

THE LOGICAL STRUCTURE OF ANSELM'S ARGUMENTS¹

IN THIS ESSAY I offer a formal analysis of Anselm's arguments for the existence of God in the *Proslogion* and in his reply to Gaunilo. I do not attempt to show here that the arguments are compelling, or that they are not. What I try to do is discover in each argument, so far as possible, a valid logical form, to exhibit the relations of the arguments to each other, and to show how they depend on certain doctrines in logic or the philosophy of logic. Anselm's arguments are far from dead, and in this paper I hope to provide a logical map, so to speak, of some ground that is still very much fought over.

The first two sections of the paper are concerned with the most famous of Anselm's arguments, the argument of Chapter 2 of the *Proslogion*. In Section I, I formulate a version of the argument in modern logical symbolism, and state the assumptions about existence and predication on which the argument seems to me to depend. Gaunilo's criticism of Anselm was directed very largely against the ontological presuppositions of the *Proslogion* 2 argument; and in Section II I try to show how Gaunilo's famous "lost island" counterexample proves that the assumptions stated in Section I must be modified, if not rejected. In his reply to Gaunilo Anselm introduced two new arguments for the existence of God, which do not depend on assumptions about predication. I discuss one of these arguments in Section III; it seems to me to be at least a better argument than the argument of *Proslogion* 2. Analysis of this argument from the reply to Gaunilo leads to the conclusion that the crucial question about logically necessary divine existence is whether it is possible. Section IV is devoted to an analysis of Anselm's argument in the third chapter of the *Proslogion* and its relation to the other arguments.

¹ I am indebted to my wife, Marilyn McCord Adams, for helpful criticism and discussion of drafts of this paper.

I

I wish to show, first of all, that the following four propositions from Chapter 2 of Anselm's *Proslogion* can be understood as the premises and conclusion of a formally valid argument.

- (1) "There is, in the understanding at least, something than which nothing greater can be thought."
- (2) "If it is even in the understanding alone, it can be thought to be in reality also,"
- (3) "which is greater."
- (4) "There exists, therefore, . . . both in the understanding and in reality, something than which a greater cannot be thought."²

The structure of the argument is complicated, and I think the apparatus of modern quantification logic may help us to state it precisely. In my formalization of the argument, the notions of existence in the understanding, existence in reality, and comparative greatness will be expressed by predicate constants as follows.

" Ux " for " x exists in the understanding"

" Rx " for " x exists in reality"

" Gxy " for " x is greater than y "

I have found it necessary also to introduce the notion of a *magnitude*, as follows.

" Qxy " for " x is the magnitude of y "

I interpret "can be thought" as meaning "is logically possible," which I think is accurate enough for the purposes of the argument, and I express this notion by means of the possibility operator,

"M," for "it can be thought that" or "it is logically possible that."

The property of being something than which nothing greater

² I am responsible for the translation of quotations from Anselm and Gaunilo in this essay. Latin text established by F. S. Schmitt and reprinted in M. J. Charlesworth's edition, translation, and commentary, *St. Anselm's Proslogion* (Oxford, 1965).

can be thought will be expressed as the property of having a magnitude such that it is not possible for anything to have a greater magnitude. This property is so complex that the argument will be easier to follow if we define a function " $\phi(x,m)$ " as follows.

$$"\phi(x,m)" = df. "Qmx \& \sim M(\exists y)(\exists n)(Gnm \& Qny)"$$

Now we can write " $(\exists m)(\phi(x,m))$ " for " x is something than which nothing greater can be thought."

The four propositions which I have quoted from Anselm will be symbolized as follows.

- (1) $(\exists x)(\exists m)(Ux \& \phi(x,m))$
- (2) $(x)(m)([Ux \& \phi(x,m)] \supset M Rx)$
- (3) $(x)(m)([\phi(x,m) \& \sim Rx] \supset \sim M \sim [Rx \supset (\exists n)(Gnm \& Qnx)])$
- (4) $\therefore (\exists x)(\exists m)(Ux \& Rx \& \phi(x,m))$

The symbolization of the third proposition calls for some explanation. All Anselm said was "which is greater." This could be taken to mean that anything which exists in reality is greater than anything which does not; this is a claim to which Anselm would probably have assented. But it could also mean just that the being under discussion (that than which nothing greater can be thought) would be greater if it existed than if not; this is all the argument requires, and I am assuming this minimal interpretation in symbolizing the argument. On this interpretation Anselm is claiming that if such a being does not exist in reality, it *would* be greater if it did exist in reality. The counterfactual conditionality of this claim poses another problem for us in symbolizing the argument. I have taken Anselm to be committed to the view that the reality of an unsurpassably great being *logically implies* its having a magnitude greater than it has if it does not exist in reality.

Anselm's argument is a *reductio ad absurdum*. "If, therefore, that than which a greater cannot be thought is in the understanding alone, that very thing than which a greater cannot be thought is something than which a greater can be thought. But certainly this cannot be."³ In my formalization I follow the same *reductio*

³ *Proslogion*, ch. 2.

strategy. The denial of what is to be proved is introduced as a premise, which is later removed by conditionalization with a self-contradictory consequent. The rules and conventions of quantification logic followed in the formalization are mainly those of Quine's *Methods of Logic*, though a modal operator is used in ways of which Quine would not approve.⁴

*	(1)	$(\exists x)(\exists m)(Ux \ \& \ \phi(x, m))$	Premise
*	(2)	$(x)(m)([Ux \ \& \ \phi(x, m)] \supset M Rx)$	Premise
*	(3)	$(x)(m)([\phi(x, m) \ \& \ \sim Rx] \supset \sim M \sim [Rx \supset (\exists n)(Gnm \ \& \ Qnx)])$	Premise
*	(4)	$Ua \ \& \ \phi(a, b)$	(1), EI (<i>a, b</i>)
*	(5)	$[Ua \ \& \ \phi(a, b)] \supset M Ra$	(2), UI
*	(6)	$M Ra$	(4), (5), TF
*	(7)	$[\phi(a, b) \ \& \ \sim Ra] \supset \sim M \sim [Ra \supset (\exists n)(Gnb \ \& \ Qna)]$	(3), UI
*	(8)	$\sim Ra \supset \sim M \sim [Ra \supset (\exists n)(Gnb \ \& \ Qna)]$	(4), (7), TF
**	(9)	$\sim Ra$	Premise
**	(10)	$\sim M \sim [Ra \supset (\exists n)(Gnb \ \& \ Qna)]$	(8), (9), TF
**	(11)	$M (\exists n)(Gnb \ \& \ Qna)$	(6), (10), modal inference
**	(12)	$(\exists y) M (\exists n)(Gnb \ \& \ Qny)$	(11), EG
**	(13)	$M (\exists y)(\exists n)(Gnb \ \& \ Qny)$	(12), see below
**	(14)	$Ua \ \& \ Qba \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)$	(4), definition of " $\phi(,)$ "
**	(15)	$M (\exists y)(\exists n)(Gnb \ \& \ Qny) \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)$	(13), (14), TF
*	(16)	$\sim Ra \supset [M (\exists y)(\exists n)(Gnb \ \& \ Qny) \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)]$	(9), (15), Conditionalization
*	(17)	Ra	(16), TF

⁴ Quine objects to the occurrence of modal operators within the scope of a quantifier. See his "Reference and Modality" (Essay VIII in his *From a Logical Point of View* [Cambridge, Mass., 1953]) and "Three Grades of Modal Involvement" (Essay XIII in his *The Ways of Paradox and Other Essays* [New York, 1966]). I am not persuaded that Quine's objection is correct, but I do not intend to discuss the issue here.

- * (18) $Ua \ \& \ Ra \ \& \ \phi(a, b)$ (4), (17), TF
 * (19) $(\exists x)(\exists m)(Ux \ \& \ Rx \ \& \ \phi(x, m))$ (18), EG

I think the inferences in this argument are clearly correct. With two exceptions they are justified by generally accepted rules of propositional and predicate logic. The inference of step (11) from steps (6) and (10) has the form “ $\sim M \sim (p \supset q)$ and $M p$; therefore $M q$ ”; this seems to me intuitively to be a valid form, and I believe it could be justified in any system of modal logic that would be likely to be used in this context. The inference of (13) from (12) is an instance of the principle that “ $M (\exists x)\psi(x)$ ” follows from “ $(\exists x) M\psi(x)$.” This principle is commonly accepted in systems of modal logic with quantifiers—and rightly so. For surely if something is such that it is possible that *it* should satisfy a certain function, then it is possible that *something* should satisfy that function. If there is a good reason for rejecting the *Proslogion 2* argument, it must be based on some objection to one or more of the premises.

I will not attempt to discuss here all of the objections which have been or could be raised against the premises. For instance, I will not consider all the possible reasons for denying that a thing could be any better or greater if it existed than if it did not. What I do want to discuss are certain general principles about existence and predication which are presupposed in the formulation and assertion of the premises of the *Proslogion 2* argument. It is because of its dependence on these presuppositions, which may plausibly be said to belong to the field of ontology, that I think this argument deserves its traditional designation as “ontological.” The following assumptions about existence and predication seem to be involved in the argument.

(i) Predication does not presuppose real existence. That is, a thing can have properties, and can be the subject of true predications, without existing in reality. (Anthony Kenny has shown, in recent publications,⁵ that this is also presupposed in Descartes’s

⁵ Anthony Kenny, *Descartes* (New York, 1968), ch. 7; a slightly longer version, with discussion by others, and Kenny’s replies, in *Fact and Existence*, ed. by Joseph Margolis (Oxford, 1969), pp. 18-62.

ontological argument, and that Descartes was aware of the presupposition—more clearly aware of it, I think, than Anselm was.) In terms of the predicate calculus used in my formalization of the argument, this means that the universe of discourse over which the variables range is not restricted to things that exist in reality. Obviously, if the universe of discourse were assumed to include only real things, the first step of the argument could not without circularity be asserted as a premise. The philosopher who is most often mentioned as having held the position that predication does not presuppose real existence is Alexius Meinong. According to him, when I think about something which does not really exist, what I am thinking about is something which not only has properties even though it does not really exist, but also would have properties even if no one ever thought about it. A thing need not exist either in the understanding or in reality to be included in Meinong's universe of discourse. Anselm's formulation of his argument, however, is consistent with the supposition that his universe of discourse is restricted to things which either exist in reality or are actually thought about.

(ii) There is another respect in which Anselm is less liberal than Meinong. He clearly assumes that the universe of discourse includes no object with contradictory predicates, whereas Meinong admits such objects (for example, the round square). If it were not assumed that self-contradictory objects are excluded from the universe of discourse, Anselm's *reductio ad absurdum* argument would collapse. For even if unreality and being a being than which a greater cannot be thought are inconsistent properties, it would not follow that the same object could not possess both.

(iii) A thing which exists in the understanding truly possesses all the properties which are contained or implied in its concept or definition. If we form a consistent description or conception of something, then whether or not it exists in reality, there is something (which at least exists in the understanding) which truly has all the properties which are included or implied in the description or conception. This appears to be presupposed for the justification of premise (1). From the fact that we understand the conception of a being than which nothing greater can be thought, Anselm infers that there is, in the understanding at least, something

which has the property of being something than which nothing greater can be thought.

(iv) One and the same thing can exist both in the understanding and in reality. Or perhaps it would be better to say that its properties are not qualified as had in the understanding or had in reality; they are simply had. At any rate, the argument, as stated so far, does not use any apparatus for qualifying properties as had in the understanding or had in reality. And it is essential to the argument to assume both that the same thing can exist in the understanding and in reality, and that it must have at least the same *defining* properties in reality as in the understanding.

(v) Existence and nonexistence in reality and existence in the understanding are predicates or properties, and it is legitimate to treat them formally in the same way as other predicates. This is obviously assumed in the argument; and at least it cannot fairly be objected that existence in reality is already expressed by the particular or "existential" quantifier, and therefore ought not to be treated as a predicate. For existence in reality is not expressed by the "existential" quantifier, if it is assumed that the universe of discourse is not restricted to things that exist in reality.

It will be convenient, though perhaps inelegant, to refer to this set of assumptions about existence and predication as "assumptions (*i-v*)."

II

The importance of Anselm's ontological assumptions was not overlooked by his first critic, Gaunilo. Indeed, almost the whole of Gaunilo's little essay can be read as a discussion of the *Proslogion* 2 argument in terms of its ontological presuppositions. He seems to hold that one is not obliged to admit that any subject of predication truly has a given property unless one is first persuaded that such a being exists in reality.⁶

Gaunilo's most famous argument is also, in my judgment, his best. The counterexample of the lost island shows quite clearly that assumptions (*i-v*) must be rejected or at least modified.

⁶ "On behalf of the Fool," chs. 5-6.

We can form a consistent description of an island, including in the description profitable and delightful features which are not in fact possessed by any island or country known to man. We can also include in the description the property of being the most excellent of all lands or countries. When I hear and understand this description, there must, according to assumptions (*i-v*), exist in my understanding an island which truly has all the properties contained or implied in the description. But then, Gaunilo claims, this island must, on Anselm's principles, exist in reality too. For suppose it does not; then "whatever other land exists in reality, will be more excellent than it."⁷ Thus the island which exists in my understanding will be both the most excellent (because that is contained in its description) and not the most excellent, which is impossible. In this way, Gaunilo suggests, something which surely does not in fact exist could be proved to exist if Anselm's assumptions about existence and predication were accepted.

I shall attempt a symbolization of this argument, so that the extent of the similarity between it and the *Proslogion* 2 argument may appear more clearly. Three new predicate symbols are required.

"*I**x*" for "*x* is an island"

"*L**x*" for "*x* is a land or country"

"*P**x*" ascribes to *x* the profitable and delightful features attributed by legend to the lost island. The proof follows the *reductio ad absurdum* pattern of *Proslogion* 2, but is shorter.

*	{	(1) $(\exists x)(Ux \ \& \ Ix \ \& \ Px \ \& \ \sim (\exists y)$ $(Ly \ \& \ Gyx))$	Premise
*	{	(2) $(\exists x)(Lx \ \& \ Rx)$	Premise
*	{	(3) $(x)(y)([Lx \ \& \ Rx \ \& \ Iy \ \& \ \sim Ry] \supset Gxy)$	Premise
*	{	(4) $Ub \ \& \ Ib \ \& \ Pb \ \& \ \sim (\exists y)(Ly \ \& \ Gyb)$	(1), EI (<i>b</i>)
*	{	(5) $La \ \& \ Ra$	(2), EI (<i>a</i>)
*	{	(6) $(La \ \& \ Ra \ \& \ Ib \ \& \ \sim Rb) \supset Gab$	(3), UI

⁷ *Ibid.*, ch. 6.

* (7)	$\sim Rb \supset Gab$	(4), (5), (6), TF
** (8)	$\sim Rb$	Premise
** (9)	Gab	(7), (8), TF
** (10)	$La \ \& \ Gab$	(5), (9), TF
** (11)	$(\exists y)(Ly \ \& \ Gyb)$	(10), EG
** (12)	$(\exists y)(Ly \ \& \ Gyb) \ \& \ \sim (\exists y)(Ly \ \& \ Gyb)$	(4), (11), TF
* (13)	$\sim Rb \supset [(\exists y)(Ly \ \& \ Gyb) \ \& \ \sim (\exists y)(Ly \ \& \ Gyb)]$	(8), (12), Conditionalization
* (14)	Rb	(13), TF
* (15)	$Ub \ \& \ Rb \ \& \ Ib \ \& \ Pb \ \& \ \sim (\exists y)(Ly \ \& \ Gyb)$	(4), (14), TF
* (16)	$(\exists x)(Ux \ \& \ Rx \ \& \ Ix \ \& \ Px \ \& \ \sim (\exists y)(Ly \ \& \ Gyx))$	(15), EG

The principal departure here from the pattern of the *Proslogion* 2 argument is that whereas Anselm spoke of a being whose greatness could not *possibly* be surpassed, Gaunilo speaks only of an island to which no country is *in fact* superior. Because of this difference, it is not necessary to use the concept of a magnitude in formulating the lost-island argument, and no possibility or necessity operator enters into the symbolization. Anselm criticized Gaunilo severely for having written as if God were characterized in the *Proslogion* as the greatest of all beings rather than as a being than which a greater cannot be thought. So far as I can see, this criticism is the only reason that Anselm gives for rejecting the lost-island counterexample.⁸ Anselm's complaint is in large measure justified. The difference between the two concepts is important in some connections, and Gaunilo does not make it clear that he grasps the distinction.

But it is not at all obvious that this failure on Gaunilo's part vitiates the lost-island counterexample. Although it does not have exactly the same form as Anselm's argument, the lost-island argument seems to be well constructed for the purpose of deriving an absurd conclusion from assumptions (*i-v*). In particular, the first premise of the lost-island argument does seem to be justi-

⁸ For Anselm's rejection of the lost-island counterexample, see the beginning of ch. 3 and the end of ch. 5 of his reply to Gaunilo; in relation to the latter, see the rest of ch. 5.

fiable, if it is true that whenever we understand a consistent description, there exists in the understanding something which has all the properties contained in the description. The second premise, that some land or country exists in reality, is obviously true. The third premise of the lost-island argument appears to have been accepted by Anselm. It expresses a more sweeping claim about the superiority of the real to the unreal than is found in the third premise of my formulation of the *Proslogion* 2 argument. But Anselm raised not a murmur of protest when Gaunilo in effect attributed to him the still more sweeping assumption that *whatever* exists in reality is greater than *anything* that does not.⁹ And the conclusion, that the lost island exists in reality, is validly implied by the three premises. Thus it was far from unreasonable for Gaunilo to suggest that the reality of the lost island could be proved on the basis of Anselmian assumptions.

But Gaunilo's counterexample is much more complicated than it needs to be, if its purpose is to discredit assumptions (*i-v*). A simpler form of counterexample was suggested by Caterus when he argued that the (real) existence of a lion could be proved from the concept (really) *existent lion* in the same way as Descartes had proved the existence of God from the concept of God.¹⁰ According to assumption (*v*), real existence is a property or predicate and can legitimately be treated as such in descriptions. And according to assumptions (*i-iii*), for every consistent description that is understood, there is some subject of which all the properties contained or implied in that description can be truly predicated. Scandalous conclusions can easily be drawn from these assumptions, along the lines suggested by Caterus. "Really existent lion," "really existent unicorn," and "really existent golden mountain" are consistent descriptions, which are understood. Therefore there exist, in the understanding at least, subjects which have all the properties contained or implied in these descriptions. These subjects must be, of course, a lion, a unicorn, and a golden mountain; and they must all really exist.

⁹ In ch. 1 of "On Behalf of the Fool."

¹⁰ In the First Objections to Descartes's *Meditations* (pp. 7-8 in vol. II of the Haldane and Ross translation of *The Philosophical Works of Descartes* [New York, 1955]).

$$(\exists x)(Ux \ \& \ x \text{ is a lion} \ \& \ Rx)$$

$$(\exists x)(Ux \ \& \ x \text{ is a unicorn} \ \& \ Rx)$$

$$(\exists x)(Ux \ \& \ x \text{ is a mountain} \ \& \ x \text{ is golden} \ \& \ Rx)$$

These propositions seem to be reached by substantially the same reasoning as the first premise of the *Proslogion* 2 argument, but real existence is already asserted in them; no further premises are needed. Assumptions (*i-v*) thus provide us with a short way to prove the real existence of anything we can think of whose description is consistent. Clearly that set of assumptions must be rejected or significantly altered.

There is more than one alteration which might be proposed to meet the objection. Descartes held, in effect, that it is only of *simple* concepts that we are entitled to assume that they are satisfied by some subject of predication.¹¹ This strikes me as a somewhat *ad hoc* modification of assumptions (*i-v*). It also seems to me doubtful whether Descartes's concept of God really is simple in a way in which the concept *existent lion* is not. Perhaps the following approach would be more promising for Anselm.

The problem which Gaunilo and Caterus have spotted in assumptions (*i-v*), we may say, is that this set of assumptions permits us to prove, from concepts alone, things which obviously are not conceptual truths. This fault might be at least partially remedied by a modification of the assumption that if a description which is understood is consistent, there must be something (real or unreal) which truly has all the properties contained or implied in the description. The application of this assumption is to be restricted to descriptions which are meant to be understood as containing only properties which belong *necessarily* to their common subject. A description which is understood to contain properties which belong contingently to their subject need not be assumed to be satisfied by anything, real or unreal.

Anselm's argument in *Proslogion* 2 seems to survive this restriction. For though Anselm does not explicitly say there that the property of unsurpassable greatness belongs to its subject necessarily, surely he assumes that it does, and would not object to

¹¹ See the discussion of this point by Kenny, Malcolm, and Sosa in *Fact and Existence*, ed. by Margolis, pp. 18-62.

being understood in that sense. But the restriction seems to dispose of the existent lion, the existent unicorn, and the existent golden mountain. In order to satisfy our new requirement, they would have to be conceived of as the *necessarily existent* lion, the *necessarily existent* unicorn, and the *necessarily existent* golden mountain. But whereas it seems clear that the descriptions "existent lion," "existent unicorn," and "existent golden mountain" are consistent, it is by no means clear, and perhaps not even plausible, that the descriptions "necessarily existent lion," "necessarily existent unicorn," and "necessarily existent golden mountain" are consistent and describe things that are logically possible. And Anselm has made no commitment to admit the inconsistent or logically impossible into his universe of discourse.

Similarly, a description of the form "island which is *P* and the best of all lands or countries" may plausibly be taken to describe something logically possible, but the same cannot be said for a description of the form "island which is *necessarily P* and *necessarily* the best of all lands or countries." Thus our restriction seems likely to rid Anselm of Gaunilo's lost island.

It is not clear that this modification of assumptions (*i-v*) eliminates all the existence proofs that one might want to eliminate. In particular, the existence of an *F* can still be proved, for any value of *F*, if "necessarily existing *F*" is a consistent description. But that is probably as it ought to be. For quite independently of any doctrines about predication, there is reason to suppose that if it is logically possible that the existence of a certain thing is logically necessary, then that thing does exist; this will be shown in the next section of this paper, with particular reference to the necessary existence of God.

I cannot conclude the present section with any triumphant vindication of Anselm. I certainly have not proved, nor have I attempted to prove, that the ontological assumptions needed for the *Proslogion* 2 argument can be justified. I have discussed only one of several objections that are often raised against the doctrine that things which do not really exist can be subjects of true predication.¹² Even the reply which I have suggested to the lost island

¹² See *ibid.*, the contributions by Kenny and Bernard Williams, for additional objections.

objection may give rise to further problems which have not been explored here. And if all apparent difficulties internal to the doctrine can be resolved, we might still wonder whether there are compelling reasons to accept it rather than some alternative way of thinking about predication. At least in the present state of philosophical research, it seems to me that one would find it advantageous to free one's arguments from dependence on such controversial ontological assumptions.

III

Apparently it seemed so to Anselm, too; for in the first chapter of his reply to Gaunilo, having noted that Gaunilo refuses to accept the principle that what is understood exists in the understanding, Anselm advances two arguments which, so far as I can see, depend neither on that principle nor on the assumption that predication does not presuppose real existence. Both of them are stated as arguments for the proposition that if something than which a greater cannot be thought can even be thought to exist, it must exist.¹³ If we add to this conclusion the premise that such a being can at least be thought to exist, we can draw the further conclusion that it does exist. Anselm obviously expects his readers to supply this additional premise and draw the further conclusion.

I shall not discuss the first of the two arguments. It turns on issues in the philosophy of time, rather than the philosophy of logic.¹⁴ That fact sets it apart from the arguments with which this essay is principally concerned.

The second argument is stated in the following words, and may be divided in three steps.

¹³ "Exist," in most contexts in these arguments from the reply to Gaunilo, obviously means "exist in reality."

¹⁴ The argument to which I am referring is the one which is expressed in the following passage, which I divide in four steps. "[1] For that than which a greater cannot be thought cannot be thought to exist except without a beginning. [2] But whatever can be thought to exist and does not exist can be thought to exist with a beginning. [3] Therefore it is not the case that that than which a greater cannot be thought can be thought to exist and does not exist. [4] Therefore if it can be thought to exist, of necessity it does exist" (pp. 168-170 in Charlesworth's text).

- (1) "For no one who denies or doubts that there exists something than which a greater cannot be thought denies or doubts that if it did exist, its nonexistence, either in actuality or in the understanding, would be impossible. For otherwise it would not be that than which a greater cannot be thought."
- (2) "But as to whatever can be thought and does not exist—if it did exist, its nonexistence, either in actuality or in the understanding, would be possible."
- (3) "Therefore if that than which a greater cannot be thought can even be thought, it cannot be nonexistent."¹⁵

This is one of the most ingenious and fascinating of Anselm's arguments. In its Anselmian form it turns on complex and partly counterfactual conditionals. And it is very difficult to tell whether the argument is sound, or even whether it is formally valid, because the logic of counterfactual conditionals is so obscure. It is possible, however, to get an argument with a clearer logical structure if we look behind the counterfactual conditionals for more basic assumptions on which steps (1) and (2) may plausibly be supposed to rest.

In the first step Anselm makes a claim which has the following conditional structure. (I use "g" as a propositional *constant* to represent "God exists" or "There exists a being than which nothing greater can be thought.")

(Even) if it is false that g, (still) if it were true that g, it would not be possible that not-g.

Anselm justifies this claim by observing that anything which did exist but whose nonexistence was possible "would not be that than which a greater cannot be thought." Evidently this first step is based on an assumption about the concept of a being than which nothing greater can be thought. Anselm believes it follows from that concept, and is therefore a necessary truth, that if such a being exists at all its existence is logically necessary. This belief can be symbolized as follows, if we use "N" as a necessity operator

¹⁵ P. 170 in Charlesworth's text.

with the meaning "it is logically necessary that" or "it cannot be thought that not."

$$N(g \supset Ng)$$

From this proposition it certainly does follow that even if it is false that g , still it would be impossible that not- g (that is, necessary that g) if it were true that g .

In the second step Anselm claims, in effect, that the following is true about every affirmative existential proposition " p " (and therefore, by implication, about " g ").

If it is possible but false that p , then if it were true that p it would still be possible that not- p .

Anselm offers no justification for this claim. But we may observe that some systems of modal logic contain a principle from which it follows that what Anselm asserts here about affirmative existential propositions is true about all propositions. The principle to which I refer is sometimes called Brouwer's Axiom, and can be expressed in the form

$$\sim p \supset NM \sim p.$$

By substituting " $\sim p$ " for " p " in this formula, and applying the rule of double negation, we could get " $p \supset NM p$," which is an alternative form of Brouwer's Axiom. But it will be more convenient for us to use the "If it is false that p " form. In either of these equivalent forms Brouwer's Axiom expresses the view that the actual state of affairs, whether positively or negatively described, would have been at least a possible state of affairs, even if any other possible state of affairs had been actual instead. From this view it obviously follows that if it is false (though possible) that p , then even if it were true that p it would still be possible that not- p —which is what Anselm asserts in step (2) about affirmative existential propositions.

I do not mean to suggest here that Brouwer's Axiom is a logical principle of undoubted validity; it is in fact a somewhat controversial principle. We shall return to that point. But first I want to offer a formal proof that Anselm's conclusion, " $M g \supset g$," does follow from " $N(g \supset N g)$ " in a system of modal logic which con-

tains Brouwer's Axiom. There are at least two known systems of modal propositional calculus in which the proof that I shall give would be formally correct. One of these is the very widely used system S₅, in which Brouwer's Axiom is not normally used as an axiom but is provable as a theorem. The other is a somewhat weaker system, called "the Brouwersche System" by Saul Kripke, of which Brouwer's Axiom is a characteristic axiom.¹⁶

It will simplify the proof if possibility operators are eliminated in favor of necessity operators. ("N" is equivalent to " $\sim M \sim$," and "M" to " $\sim N \sim$.".) Thus Brouwer's Axiom will be stated as " $\sim p \supset N \sim N p$ " instead of " $\sim p \supset NM \sim p$," and the conclusion as " $\sim N \sim g \supset g$ " instead of " $M g \supset g$." In addition to Brouwer's Axiom, the argument will appeal to several rules of inference which are part of the classical nonmodal propositional calculus as well as of the relevant modal systems, and to the following two modal principles.

$$(T_1) N(p \supset q) \supset N(\sim q \supset \sim p)$$

$$(T_2) N(p \supset q) \supset (N p \supset N q)$$

(T₁) is provable as a theorem in the two modal systems mentioned above; and (T₂), which is also contained in them, is commonly treated as one of their axioms. Here is the proof:

(1) $N(g \supset N g)$	Premise
(2) $N(g \supset N g) \supset N(\sim N g \supset \sim g)$	Substitution in (T ₁)
(3) $N(\sim N g \supset \sim g)$	(1), (2), <i>modus ponens</i>
(4) $N(\sim N g \supset \sim g) \supset (N \sim N g \supset N \sim g)$	Substitution in (T ₂)
(5) $N \sim N g \supset N \sim g$	(3), (4), <i>modus ponens</i>
(6) $\sim g \supset N \sim N g$	Substitution in Brouwer's Axiom

¹⁶ Kripke, "Semantical Analysis of Modal Logic I: Normal Propositional Calculi," *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, IX (1963), 67-96. See also C. I. Lewis and C. H. Langford, *Symbolic Logic*, 2nd ed. (New York, 1959), p. 498, on the relation of Brouwer's Axiom to S₅.

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|-------------------------------|--|
| (7) $\sim g \supset N \sim g$ | (5), (6), hypo-
thetical syl-
logism |
| (8) $\sim N \sim g \supset g$ | (7), transporta-
tion, double
negation |

This argument can easily be extended to form a proof of the existence of God, by the addition of the premise (which Anselm obviously meant his readers to supply) that the existence of a being than which nothing greater can be thought is at least possible. The argument continues as follows.

- | | |
|---------------------|-------------------------------|
| (9) $\sim N \sim g$ | Premise |
| (10) $\therefore g$ | (8), (9), <i>modus ponens</i> |

This proof is similar to an argument presented in modern modal logical notation by Charles Hartshorne.¹⁷ The chief difference is that Hartshorne's argument uses, instead of Brouwer's Axiom, the principle " $\sim N p \supset N \sim N p$," which is a characteristic axiom of S₅ but is not contained in the Brouwersche System at all.¹⁸ This difference is formally interesting, but practically of little or no importance; for there is not likely to be any good reason for accepting one of the two principles in the present context which would not also be a good reason for accepting the other.

Hartshorne calls his argument a "modal argument" for the existence of God; and that also seems a good name for the similar argument introduced by Anselm in Chapter 1 of his reply to Gaunilo, because it is an argument that lends itself to formal paraphrase as primarily an argument in modal propositional logic. I think it is better not to call these arguments "ontological" because, unlike the argument of *Proslogion* 2, they need not depend on any assumptions at all about the relation of existence to predication. They do not presuppose that things which do not really

¹⁷ *The Logic of Perfection* (La Salle, Ill., 1962), pp. 50-51.

¹⁸ Actually Hartshorne uses an axiom equivalent to " $N(\sim N p \supset N \sim N p)$ "; the initial necessity operator is not needed here. But that is a minor point.

exist can have predicates. They do not presuppose that existence, or existence in reality, is a predicate, nor even that necessary existence is a predicate. For their structure does not depend on predicate logic at all, but only on modal and nonmodal propositional logic. Obviously it is a great advantage to Anselm to be able to dispense with those controversial assumptions about predication.

Perhaps it will be objected, however, that Anselm's modal argument has compensating disadvantages of its own in the doctrines of modal logic which it assumes. I have already noted that there has been controversy over the question whether it is legitimate to use systems of modal logic containing Brouwer's Axiom. Doubtless some of this controversy is due to a failure of philosophers at first to appreciate that the question "What is the correct system of modal logic?" is misconceived. The modal terms "necessary" and "possible" have more than one sense, and are sometimes used rather vaguely. Whether a given system of modal logic is valid or not depends on the interpretation that is assigned to its modal operators.

For instance, the interpretation of the necessity operator "N" might be determined, or partially determined, by any one of the following semantical rules.

- (R₁) "N *p*" is true if and only if "*p*" is true about all possible worlds.
- (R₂) "N *p*" is true if and only if "*p*" is true solely by virtue of the meaning rules of our language being what they are.
- (R₃) "N *p*" is true if and only if "*p*" is logically provable.

Saul Kripke has shown that if (R₁) is assumed, and if it is further assumed that just the same worlds would be possible no matter what world were actual, S₅ (and therefore also the Brouwersche System) must be accepted as valid. I think there is also good reason to believe that if the interpretation of "N" is determined by (R₂), S₅ and the Brouwersche System are valid. In particular, it seems that if it is not the case that a certain proposition is true solely by virtue of the meaning rules of our language being what they are, that itself is something that could not be otherwise, the

meaning rules of our language being what they are. Therefore, according to (R₂), " $\sim N p \supset N \sim N p$," the controversial axiom of S₅, is valid; and if it is, so is Brouwer's Axiom, " $\sim p \supset N \sim N p$." On the other hand, if the interpretation of "N" is determined by (R₃), it is plausible to suppose that " $\sim N p \supset N \sim N p$ " is not valid; for what is not logically provable may not be provably unprovable. But " $N p \supset N N p$," a characteristic axiom of the modal system S₄, presumably would be valid, since what can be proved can thereby be proved provable. And it is known that if " $N p \supset N N p$ " is valid and " $\sim N p \supset N \sim N p$ " is not valid, Brouwer's Axiom is not valid either.¹⁹

All of this bears on the modal arguments for the existence of God, as follows. We are talking about arguments which derive the conclusion "g" from the premises " $N (g \supset N g)$ " and " $\sim N \sim g$ " in some system of modal propositional calculus. I have stated such an argument, which is formally correct in S₅ and the Brouwersche System. Because the argument uses the Brouwersche Axiom, however, it is not formally correct in S₄. Indeed, it can be proved that " $[N (p \supset N p) \ \& \ \sim N \sim p] \supset p$ " is not a valid formula in S₄,²⁰ so that S₄ cannot be used to derive "g" from " $N (g \supset N g)$ " and " $\sim N \sim g$." But it seems to me extremely plausible to suppose that the interpretation of the necessity operator in the modal arguments for the existence of God can rightly be regarded as determined by (R₁) or (R₂), and that S₅ or the Brouwersche System is therefore valid in the context with which we are concerned. I do not claim that this conclusion is absolutely certain, but I think it is so plausible that the disadvantages of assuming Brouwer's Axiom in this context are rather small. They are quite inconsiderable in comparison with the disadvantages of the assumptions about existence and predication which are involved in the ontological argument of *Prosligion 2*.

¹⁹ This claim presupposes some other principles of modal logic which would not be controversial in this context. On the issues discussed in this paragraph, see the works cited in n. 16; also Saul Kripke, "A Completeness Theorem in Modal Logic," *Journal of Symbolic Logic*, XXIV (1959), 1-14; and E. J. Lemmon, "Is There Only One Correct System of Modal Logic?," *Proceedings of the Aristotelian Society*, supp. vol. XXXIII (1959), 23-40.

²⁰ This can most readily be shown by the method of semantic tableaux which is explained by Kripke in the two works cited above.

This gives us some reason to suppose that Anselm's modal argument for the existence of God is a better argument than the *Proslogion* 2 argument. The logical doctrines assumed in the modal argument are less questionable than those assumed in the ontological argument. Therefore the modal argument is at least the better argument of the two unless its premises are decidedly less plausible than those of the *Proslogion* 2 argument. And I believe the premises of the modal argument are not less plausible than the premises of *Proslogion* 2. I will not attempt to prove that this belief of mine is right, but I will develop one line of thought which may tell in its favor.

The modal argument for the existence of God, in the last form in which I stated it, has two premises.

$$\begin{aligned} N(g \supset N g) \\ \sim N \sim g \end{aligned}$$

(Modal axioms are not properly premises, but part of the logical apparatus.) The second premise says that it is possible that God exists. But what sort of God? If the rest of the argument is sound, it is a logically necessary God whose existence is here assumed possible. If God cannot exist at all unless it is necessary that God exist, then the claim that God's existence is possible implies that it is possible that it is necessary that God exists. This proposition, that it is possible that it is necessary that God exist ($\sim N \sim N g$) is what any supporter of the modal argument must defend in trying to justify the premise that God's existence is possible. An alert critic of the argument will not let him get away with less. But if the proposition " $\sim N \sim N g$ " is granted, no other premise is needed in order to prove the existence of God by a modal argument. All that is needed is Brouwer's Axiom and two truth-functional rules of inference (*modus tollens* and double negation). The proof is extremely simple.

(1) $\sim N \sim N g$	Premise
(2) $\sim g \supset N \sim N g$	Substitution in Brouwer's Axiom
(3) $\therefore \sim \sim g$	(1), (2), <i>modus tollens</i>

(4) $\therefore g$ (3), double
negation

This reasoning shows two things about the proposition " $\sim N \sim N g$." First, " $\sim N \sim N g$ " is already, in effect, a premise in Anselm's modal argument for the existence of God. Second, " $\sim N \sim N g$," all by itself, as sole premise, is sufficient for a proof of the existence of God, if the use of Brouwer's Axiom in these arguments is justified. If we are given this one premise of the modal argument, we do not need any other premise to prove the existence of God.

I will add a third point. " $\sim N \sim N g$ " is a proposition which must be maintained by anyone who holds that the existence of God is logically necessary. For if it is necessary that God exist, it is possible that it is necessary that God exist; whatever is the case is also possible. Very commonly one assumes without question that what one is trying to prove is at least logically possible. But in the present case, what is to be proved follows by a very short argument from the proposition that it is possible. The crucial question, therefore, about logically necessary divine existence is the question of possibility, the question whether it is logically possible that it is logically necessary that God exist. Other premises can properly support belief in logically necessary divine existence only in so far as they support belief in its possibility.

Let us return to the comparison of the merits of the modal and ontological arguments. This at least can be said. The *Proslogion* 2 argument has probably never been defended except as part of a program for proving the logically necessary existence of God. It could be of value for a proof of that conclusion only if it helped to make more plausible the claim that it is logically possible that God's existence is logically necessary. It seems to me very unlikely that it would help in that way. Indeed, I think it is correct to say that although the modal argument for the existence of God helps us to see that the question of possibility is the crucial question about logically necessary divine existence, neither the modal nor the ontological argument provides us with grounds for answering it.

IV

When commentators have looked in Anselm for a "second ontological argument," possibly better than the argument of *Proslogion* 2, they have commonly looked for it in a passage which I have not yet discussed, in the third chapter of the *Proslogion*. It is generally acknowledged that Anselm did not intend to present there a second, independent argument for the existence of God, but to prove an additional proposition about the being which was already proved to exist in Chapter 2. It is suggested, however, that the argument which Anselm uses for this purpose can be adapted to form an independent proof of God's existence.

In fact, there is an argument in the third chapter of the *Proslogion* which is capable of more than one interpretation and could be used to prove more than one conclusion. I will begin my discussion of it by stating a formal pattern of reasoning which is pretty much the same for all the interpretations and uses of the argument. Then I will discuss briefly three different uses to which this pattern of reasoning has been put by Anselm and his admirers.

Two premises are stated in *Proslogion* 3, in the following terms.

- (1) "For it can be thought that there exists something which cannot be thought not to exist,"

$$M (\exists x)(\sim M \sim Rx)$$

- (2) "which is greater than what can be thought not to exist."

$$(x)(m)([Qmx \& M \sim Rx] \supset \sim M \sim (y)[\sim M \sim Ry \supset (\exists n)(Gnm \& Qny)])$$

Under any interpretation of the argument a third premise is required, which is not stated in *Proslogion* 3 but must be carried over from Chapter 2. This premise says that some subject of predication has the property of being something than which nothing greater can be thought.

$$(3) (\exists x)(\exists m)(Qmx \& \sim M (\exists y)(\exists n)(Gnm \& Qny))$$

This formula might be interpreted as saying that some *really existing* subject of predication has the property of unsurpassable

greatness. Or it might be understood, like the first premise of the *Proslogion* 2 argument, as asserting only that some subject of predication, which may or may not exist in reality, has the property of unsurpassable greatness. The choice between these two interpretations determines the ontological presuppositions of the argument and is important in relation to some of its uses, but it makes no difference to the formal structure of the argument.

The argument, like that of *Proslogion* 2, is presented by Anselm in *reductio ad absurdum* form. This will be carried over into the formalization of the argument. A thesis to be refuted will be introduced as a premise which will subsequently be removed by conditionalization with a self-contradictory consequent.

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|-----|---|--|-------------------------------|
| * | { | (1) $M (\exists x)(\sim M \sim Rx)$ | Premise |
| * | } | (2) $(x)(m)([Qmx \ \& \ M \sim Rx] \supset \sim M \sim (y)[\sim M \sim Ry \supset (\exists n)(Gnm \ \& \ Qny)])$ | Premise |
| ** | | (3) $(\exists x)(\exists m)(Qmx \ \& \ \sim M (\exists y)(\exists n)(Gnm \ \& \ Qny))$ | Premise |
| ** | | (4) $Qba \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)$ | (3), EI (a, b) |
| ** | | (5) $[Qba \ \& \ M \sim Ra] \supset \sim M \sim (y)[\sim M \sim Ry \supset (\exists n)(Gnb \ \& \ Qny)]$ | (2), UI |
| ** | | (6) $M \sim Ra \supset \sim M \sim (y)[\sim M \sim Ry \supset (\exists n)(Gnb \ \& \ Qny)]$ | (4), (5), TF |
| *** | | (7) $M \quad Ra$ | Premise |
| *** | | (8) $\sim M \sim (y)[\sim M \sim Ry \supset (\exists n)(Gnb \ \& \ Qny)]$ | (6), (7), TF |
| *** | | (9) $\sim M \sim [(\exists x)(\sim M \sim Rx) \supset (\exists y)(\exists n)(Gnb \ \& \ Qny)]$ | (8), see below |
| *** | | (10) $M (\exists y)(\exists n)(Gnb \ \& \ Qny)$ | (1), (9), modal inference |
| *** | | (11) $M (\exists y)(\exists n)(Gnb \ \& \ Qny) \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)$ | (4), (10), TF |
| ** | | (12) $M \sim Ra \supset [M (\exists y)(\exists n)(Gnb \ \& \ Qny) \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny)]$ | (7), (11), conditionalization |
| ** | | (13) $\sim M \sim Ra$ | (12), TF |
| ** | | (14) $Qba \ \& \ \sim M (\exists y)(\exists n)(Gnb \ \& \ Qny) \ \& \ \sim M \sim Ra$ | (4), (13), TF |

** (15) $(\exists x)(\exists m)(Qmx \ \& \ \sim M(\exists y)(\exists n)$
 $(Gnm \ \& \ Qny) \ \& \ \sim M \ \sim Rx)$ (14), EG

The inference of step (10) from steps (1) and (9) has the form “ $\sim M \sim (p \supset q)$ and $M p$; therefore $M q$,” which, as I said in Section I, seems to me intuitively to be valid under any interpretation of the possibility operator which would fit the present context. It is obvious that (9) follows validly from (8) when we realize that (8) means “It is necessary that if anything exists necessarily, then *it* has a magnitude greater than *b*,” and (9) means “It is necessary that if anything exists necessarily then *something* has a magnitude greater than *b*.”

We turn now to the three uses of this pattern of reasoning.

(A) In *Proslogion* 3 Anselm seems to intend to prove an additional proposition about a being who has already, in the second chapter, been proved to exist in reality. It is not said in Chapter 2 that the being of unsurpassable greatness exists necessarily. Neither is it clear that the *Proslogion* 2 argument, if successful, must establish its conclusion with logical necessity. For it depends on the claim that someone has a concept of an unsurpassably great being. And for all that is said in Chapter 2, that claim may well be only contingently true. So Anselm is trying to prove something new about the unsurpassably great being when he argues in Chapter 3 that such a being “exists so truly that it cannot even be thought not to exist.” He may be seen as building on the foundation laid in the second chapter. And in that case premise (3) of the *Proslogion* 3 argument may be interpreted as asserting the real existence that is supposed to have been proved in *Proslogion* 2. Under this interpretation, of course, the Chapter 3 argument inherits all the weaknesses of whatever argument it depends on for the justification of its existential premise.

(B) It is easy to see how the *Proslogion* 3 argument may be regarded as being, in its own right, a distinct argument for the existence of God. Its conclusion states that something has the property of unsurpassable greatness and necessarily has real existence. This conclusion clearly implies that such a being really exists; for what necessarily has real existence does have real existence.

But if the argument of *Proslogion* 3 is to be an additional proof of the real existence of God, it cannot without circularity have a premise which presupposes or explicitly asserts the real existence of God. Hence, the universe of discourse for the formalization of the argument must not be understood as restricted to things that really exist. For if the universe of discourse were restricted to real existents, premise (3) would already assert the real existence of a being than which a greater cannot be thought. This means that if the *Proslogion* 3 argument is to be interpreted as an independent and noncircular argument for the reality of God, it must be understood as resting on the assumption that predication does not presuppose real existence. Considered as a "second" proof of the real existence of God, the argument of *Proslogion* 3 is indeed an "ontological" argument, involving substantially the same assumptions about the relation of existence to predication as are involved in *Proslogion* 2.

(C) As I have already tried to show, Anselm assumes, in the first chapter of his reply to Gaunilo, that it is a necessary truth that if God exists at all, His existence is necessary: $N(g \supset N g)$. The argument of *Proslogion* 3 can be seen as providing reasons for this assumption. If the argument is used for this purpose, it is most convenient to regard the universe of discourse as restricted to real existents, so that the existential or particular quantifier " $(\exists x)$ " can be read as "There exists (in reality) an x such that." A sixteenth step is added to the argument in which premise (3) is removed by conditionalization.

* (16) $(\exists x)(\exists m)(Qmx \ \& \ \sim M(\exists y)(\exists n)(Gnm \ \& \ Qny)) \supset (\exists x)(\exists m)(Qmx \ \& \ \sim M(\exists y) \ (\exists n)(Gnm \ \& \ Qny) \ \& \ \sim M \ \sim Rx)$ (3), (15), conditionalization

Now we have a formally valid argument from premises (1) and (2) for the conclusion that *if* there exists (in reality) something than which nothing greater can be thought, *then* there exists (in reality) something than which nothing greater can be thought, and which cannot be thought not to exist. This argument does not depend on the denial of the doctrine that predication presupposes real existence; for its variables range over real existents only. Neither do its premises assert or imply the real existence of an unsur-

passably great being, since it no longer has an existential premise.²¹ If (1) and (2) are necessary truths, as Anselm presumably believed them to be, the argument shows that (16), which follows from them, is also a necessary truth.

Even when we prefix a necessity operator to (16), however, we do not yet have " $\text{N}(g \supset \text{N}g)$ " but something of a quite different form. The antecedent of (16) says that a certain description is satisfied. But the consequent does not say it is necessary that that description is satisfied. It says that the description is in fact satisfied, and by an individual whose real existence is conceptually necessary. This might still leave open the possibility that in some possible world the description would not be satisfied, though the individual in question would of course exist, but without satisfying it. In order to rule out this possibility, Anselm would have to assume that any individual which is unsurpassably great must possess that property by conceptual necessity. That is, unsurpassable greatness must be an essential property of the individual in question, a property without which He could not be the individual He is. I am sure that Anselm does hold this (see *Monologion*, Chapters 16-17). With this additional assumption, the argument of *Proslogion* 3 can be regarded as giving reasons, which may have been Anselm's original reasons, for accepting the proposition " $\text{N}(g \supset \text{N}g)$."

I shall not attempt here to evaluate in detail this argument for " $\text{N}(g \supset \text{N}g)$ "; but there is one more point that I do want to make about it. " $\text{N}(g \supset \text{N}g)$ " might be an important premise for someone who was trying to prove the *nonexistence* of God by a modal argument. J. N. Findlay once offered a modal *disproof* of the existence of God,²² which seems to have the following general form.

- (i) God cannot exist (in reality) at all unless He exists (in reality) by conceptual necessity.

²¹ As the argument now stands, real existence is expressed in it by two different notations. It might be desirable when interpreting the argument in this way to replace " Rx " consistently by " $(\exists z)(z = x)$." This could be done without damage to the structure of the argument.

²² J. N. Findlay, "Can God's Existence Be Disproved?" *Mind*, LVII (1948), 176-183. The article has been reprinted several times.

- (ii) It is not possible for anything to exist (in reality) by conceptual necessity.
- (iii) Therefore it is not possible for God to exist (in reality) at all.

The point that I want to make is that anyone who used this atheistic argument could not consistently support its first premise by the argument of *Proslogion* 3. For premise (1) of the *Proslogion* 3 argument says that it is possible for something to exist in reality by conceptual necessity. And intuitively it seems highly plausible that that should be a premise in an argument for the proposition that an unsurpassably great being cannot exist (in reality) except by conceptual necessity. It would hardly be reasonable to regard a being's greatness as surpassable simply because it lacked a property which could not possibly be possessed by anything at all. But premise (ii) of the atheistic argument flatly denies that it is possible for anything to exist in reality by conceptual necessity; it is therefore inconsistent with one of the premises of the *Proslogion* 3 argument. Of course I have not proved here that the proponent of the atheistic argument cannot support its first premise at all. But at least he cannot consistently establish it by the use of the argument in *Proslogion* 3.

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