# Reading 'On Denoting' on its Centenary

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Part 1 sets out the logical/semantical background to 'On Denoting', including an exposition of Russell's views in *Principles of Mathematics*, the role and justification of Frege's notorious Axiom V, and speculation about how the search for a solution to the Contradiction might have motivated a new treatment of denoting. Part 2 consists primarily of an extended analysis of Russell's views on knowledge by acquaintance and knowledge by description, in which I try to show that the discomfiture between Russell's semantical and epistemological commitments begins as far back as 1903. I close with a non-Russellian critique of Russell's views on how we are able to make use of linguistic representations in thought and with the suggestion that a theory of comprehension is needed to supplement semantic theory.

My project is primarily expository and context setting. I also want to correct a few misunderstandings that Russell or I or others may have had. Although I flag a few issues in my own voice, I am trying, on the whole, to present my discussion and analysis in a way that is recognizably Russellian. Part 1 sets out the logical/semantical background to 'On Denoting', including an exposition of Russell's views in *Principles of Mathematics* and speculation about how the search for a solution to the contradiction Russell had discovered in Frege's logic might have moti-

[1] This paper is drawn from the course on 'On Denoting' that I have taught at UCLA for more than thirty years. I thought this was a good opportunity to produce class notes. A few months ago, when I read Alasdair Urquhart's surprising 'Introduction' to the invaluable, but very expensive, fourth volume of *The Collected Papers of Bertrand Russell* as well as Russell's unpublished papers between *Principles of Mathematics* (1903) and 'On Denoting' (1905), I came to better understand that Russell's attempts to avoid the contradiction he had found in Frege's logic were related to his worries about denoting. Urquhart's discoveries made some old views about the relation between Frege's logic and type theory relevant to Russell's project. My discussion in Part 1 mixes historical fact and speculation with logical fact and speculation. The unpublished papers before 1905 also throw light on Russell's concerns about how we *understand* language. These concerns led to his distinction between knowledge by acquaintance and knowledge by description. My discussion of these issues, in Part 2, again mixes historical fact and speculation with, in this case, semantical and epistemological fact and speculation. I am not an historian, and though I have tried to read Russell carefully, I am pitifully ignorant of the secondary literature. Even regarding the primary sources, Russell wrote more from 1900–1925 than I could read with adequate care in my lifetime. So though I write assertively, I expect scholars to find faults. I welcome correction. This paper is dedicated to Volume 4 of *The Collected Papers of Bertrand Russell*. It has benefited from the comments of Joseph Almog, C. Anthony Anderson, Benjamin Caplan, John Carriero, Timothy Doyle, Ruth Marcus, Donald A. Martin, Youichi Matsusaka, and Stephen Neale. Anderson caught one of the embarrassing errors in time for me to fix it. Martin provided useful discussion of Axiom V.

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vated a new treatment of denoting. I try to locate Russell's earlier views in relation to Frege's, and to frame the new treatment of denoting in opposition to these earlier views. In Part 2, I begin my examination of 'On Denoting' itself, the logical, semantical, and epistemological theses it proposes. I start with an analysis of Russell's use of 'denoting phrase' and follow with an extended discussion of Russell's views on knowledge by acquaintance and knowledge by description. I try to show that the discomfiture between Russell's semantical and epistemological commitments begins as far back as 1903. Part 2 completes my review of the first two paragraphs of 'On Denoting'. I hope to write a sequel.

## Part 1: Background

### *1.1 Principles of Mathematics*

#### ***1.1.1 Language as a system of representation***

It is, or should be, generally accepted that 'On Denoting' (hereafter OD) is written in opposition to Russell's own views in the chapter entitled 'Denoting' in *Principles of Mathematics* (hereafter *PoM*). That chapter presupposes Russell's view that language is a system for representing things and arrangements of things in the world. The simple elements of language stand for things and properties, and linguistically complex expressions stand for complexes of those things and properties. Russell calls the kind of thing that a *sentence*, the most important linguistically complex expression, stands for (or *expresses*, or *means*) a *proposition*. Hence, the constituents of propositions are the very things that the propositions are *about*. For example, the sentence 'I met Bertie' expresses a proposition whose constituents are me, Bertie, and the relational property *meeting*. All three of these constituents are entities to be found in the empirical world, according to Russell.[2] A proposition (and any sentence that expresses it) is true if the way the things are arranged in the world 'corresponds' to the way the things are arranged in the proposition, in the case in question, if the relation of *meeting* actually held between me and Bertie.

Propositions have a *structure*, a kind of syntax of their own. Russell often talks as if this syntax mirrored the syntax of natural language. In OD he modifies that view, as we shall see. The propositions exhibit all the ways that the objects and properties of the world can be combined in accord with this propositional syntax. What propositions there are is

[2] Of course, if the sentence is about numbers or other non-worldly entities, the propositional constituents will not be worldly, but they will still be the things the proposition is about.

 

determined by what objects and properties there are in the world. What sentences there are is determined by a narrower range of facts, including, for example, which objects and properties are of interest to the creators of the language. Not every proposition need be expressed by a sentence in an actual language. For Russell, his contemporaries, and those that preceded them, it is the realm of propositions, existing independently of language, that form the subject matter of logic.[3] One consequence of this propositions-before-language point of view is that the symbolism used in the language of logic must be developed with great care. Our ability to study the logical relations among propositions may be helped or hindered by how well the syntax of the language of logic articulates with the structure of the propositions that form its subject matter.[4]

The view that language is a system of representation for the things and states (and possible states) of the world seems natural and appealing, but it is not the only way to view language. Gottlob Frege, the great creator of modern symbolic logic and founder of Logicism, saw language as based on *thought*.[5] On Frege's picture, language is an *externalization* of, and thus a system for representing, *thought*. Frege's meanings, unlike Russell's, are elements of cognition and complexes of such ele-

[3] Others may have had a somewhat different conception of the nature of propositions, but the view that the objects of logical study are prior to language was very widespread.

[4] Russell's conception seems to pose a challenge to the interpretation of modality in so far as it is considered possible for there seems to be things other than there are. This is because his propositional functions seem not to be intensional in their domain. It may be possible to repair this difficulty without either adding merely possible objects to the domains of propositional functions (which would have caused Russell to shudder) or replacing propositional functions with relations among properties (which would be unfaithful to the notion of a propositional function) by analyzing *his* notion of a function. Russell's scepticism about modality, expressed in his unpublished 1905 paper 'Necessity and Possibility' (which can be found in *The Collected Papers of Bertrand Russell*, *Volume 4* (hereafter *CP4*)) may have prevented him from ever confronting this challenge directly.

[5] Logicism is the view that mathematical constants can be defined in pure logic, and that such definitions provide a reduction of all truths of mathematics to truths of pure logic. Historically, it seems to have been part of Russell's view that the truths of mathematics would be reduced truths of logic that would *provable* in a single all-encompassing systematization of logic. For example, on p. 4 of *PoM*, Russell describes 'the Kantian view, which asserted that mathematical reasoning is not strictly formal'. He then writes, with an excess of confidence, 'Thanks to the progress of symbolic logic, especially as treated by Professor Peano, this part of the Kantian philosophy is now capable of a final and irrevocable reputation. By the help of 10 principles of deduction and 10 other premises of the general logical nature (e.g. "implication is a relation"), all mathematics can be strictly and formerly deduced; and all the entities that occur in mathematics can be defined in terms of those that occur in the above 20 premises.' Frege is more cautious. Although he rails against loose standards of proof, he typically demands only the proof of the 'the fundamental propositions of arithmetic'. For example, in his Grundlagen der Arithmetik, he writes on page 4e, '… we are led to formulate the same demand as that which had arisen independently in the sphere of mathematics, namely that the fundamental propositions of arithmetic should be proved, if in any way possible, with the utmost rigor; for only if every gap in the chain of deductions is elimi-

 

ments.[6] Like Russell's propositions, Frege's *thoughts* precede language. Frege claims that there is a repertoire of *thoughts* common to all mankind, and thus independent of the particulars of actual languages.

Frege used the word *Sinne* ('senses') for the cognitive elements and complexes that are represented by linguistic elements and complexes. Thus, for Frege, the *sense* of a linguistic expression is what the expression represents or means.[7]

Russell spoke of thought as something psychological, and stated that his interest was in the *object* of thought (now sometimes referred to as the *content* of a thought). Russell assumed, essentially without argument, that the kind of thing that served his semantic theory, i.e., his theory of meaning for language, was also the kind of thing that served as an object of thought. So Russell also referred to the objects of thought as *propositions*, and sometimes, perhaps to emphasize the fact that the constituents of propositions are the very objects that the propositions are about, as *objective propositions*. Propositions are thus a common element connecting linguistic representation with thought, and this provides a foundation for explaining our *understanding* of language. Although sentence meanings and objects of thought are the same *kind* in Russell, he was aware that it did not follow that any proposition that could be represented linguistically could be an object of thought.[8]

Frege does not distinguish thoughts from the objects or contents of thoughts, as Russell did. Frege's *thoughts* are also 'objective', but in a dif-

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nated with the greatest care can we say with certainty upon what primitive truths the proof depends …' In any case, we now know that not all truths of mathematics can be proved in a single all-encompassing mathematical system of the kind envisaged, so Russell's requirement is too strong and should be replaced by the requirement to reduce truths to truths and proofs to proofs.

[6] I speak a bit loosely here. 'Complex' is Russell's word. It is not clear that Frege's meanings actually have constituents in Russell's sense. What seems common to the two views is that the meanings of complex expressions can be parsed into sub-meanings in a way roughly corresponding to the way in which the complex linguistic expressions can be parsed into sub-expressions, though the parsing of meanings might not exactly correspond to what a grammarian would tell us about the parsing of expressions. (Frege remarks that active and passive constructions may have the same meaning, and in 'Function and Concept' he claims that '$(x)(x^2-4x = x(x-4))$' has the same meaning as '$\lambda x(x^2-4x) = \lambda x(x-4)$' (where the $\lambda$ notation is for the course-of-values of a function as discussed below). These examples suggest that Frege's parsings may not be unique, and thus that Frege's meanings may not have a constituent structure. This I owe to Terry Parsons.

[7] The reader will have noted that my use of *italics* goes one better than Russell, mixing together in one notation, reference to expressions and to their meanings along with the traditional use for emphasis.

[8] It also doesn't follow that any object of thought could be represented through language, but Russell doesn't seem to have been interested in this.

 

ferent sense. The same *thought* can be shared. So Frege's *thoughts* are not 'psychological' in the sense of being subjective and unshareable. But they are certainly not 'objective' in Russell's sense of having worldly objects as constituents. This seems to leave Frege's *thoughts* high and dry, divorced from reality (as thought can so easily be). So Frege postulates a second kind of representation whereby the elements and complexes of cognition represent worldly things. This second kind of representation, which he calls 'Bedeutung', is dependent on worldly facts; it is not determined by thought alone. The same elements of thought, in other circumstances, could represent different objects, and so the same thought could represent a different structure of worldly elements. For Frege, it is through this second kind of representation that thoughts, and ultimately sentences, come to be true and false.[9]

It is often said that the cognitions that Frege associates with a name are in fact definite description-like in structure.[10] This would explain the relation of Bedeutung that holds between such a cognition and a worldly individual. But the explanation only works on the basis of a prior explanation of the Bedeutung relation that holds between the 'predicates' of the description-like cognition and (roughly) the classes of individuals to which they apply. This relation, which is left fairly mysterious, seems to be based on an implicit link (perhaps, identity)

[9] Russell and Frege belonged to a mutual admiration society. There was, however, much miscommunication between them. It is my belief that a prime reason for this miscommunication was that neither ever quite understood or accepted that the other's treatment of language was so fundamentally different from his own. Russell says in *PoM* that Frege's semantical system is very much like his own. And he repeats this claim frequently. Neither ever seemed to fully grasp their fundamental divergence over whether language is a system for representing things and states of the world or things and states of the mind. This miscommunication was also engendered by the fact that they use the same language to mean very different things. In their correspondence, much of which is published, one sees them frequently talking past one another with Frege trying to lay out his conceptual apparatus in careful and precise detail, and Russell responding in terms of his own conceptual apparatus, but using pretty much the same language. The difference between Russell's propositions and Frege's thoughts lies at the heart of the difference between them. But these two notions seem to have been conflated, perhaps because *thoughts* are for Frege, just as *propositions* are for Russell, expressed by sentences and the objects of mental activity. It was also a very great misfortune that Frege had chosen to use the word 'Bedeutung' for a notion close to what Russell called *denotation*. Russell translated 'Bedeutung' in the customary way as *meaning*, which he *contrasted* with denotation. What Russell meant by the English word 'meaning' was much closer to what Frege meant by 'Sinn', which Frege contrasted with *Bedeutung*. They corresponded in German, and, as far as I can tell, the translation problem never quite sunk in. Russell must have been stupefied to see Frege write, as he did on December 28, 1902, 'You could not bring yourself to believe that the truth-value is the meaning of a proposition'. To which Russell responds on December 12, 1904, '[F]or me, the *meaning* of a proposition is not the true, but a certain complex which (in the given case) is true'. This disagreement is surely a problem engendered primarily by the translation of 'Bedeutung'. The correspondence is published in Frege's *Philosophical and Mathematical Correspondence*.

[10] I'm not sure that this is invariably correct, but it *is* often said.

 

between the elements of cognition that are represented by words like 'red', 'hot', 'dog', and 'star', and the properties and relations that Russell considered worldly. We don't have presentations of the classes themselves, so it seems that we must rely on presentations of properties (which, given the actual facts, could determine the classes). The simplest hypothesis is that the cognitions in question *are* the properties and relations. An alternative hypothesis is that the cognitions in question come to be *of* (in a *third way* of representation) such properties and relations through presentations thereof. A difficulty with the first is that the cognitions themselves are supposed to be innate.[11] If Frege were to abandon the innateness claim, and accept Russell's view that we become acquainted with worldly property and relations through experience, he could bring properties and relations directly into the realm of thought. But he would then face the worry that the same property might be presented in ways that we fail to identify, for example, we may fail to recognize every presentation of the property of being a *dog* ('Dogs range in size and form from the diminutive Chihuahua to the monstrous Great Dane, and every size and shape imaginable in-between making the domestic dog, Canis familiaris, the most varied species on the face of the planet.'), and distinct properties, for example, the property of being a *planet* and the property of being a *star*, might be presented in ways that led us, mistakenly, to identify them. These errors could cause us to mistake one thought for another. This would be a serious difficulty for Frege's theory, since it was designed to *explain* errors of recognition from a standpoint that was free of them. We will return to worries about recognition.

If we put these concerns aside for now (a big IF), and suppose that the elements of cognition represented by words like 'red', 'hot', 'dog', and 'star' *are* Russellian properties (or if we close the gap between Fregean senses of these words and Russellian properties in some other way), Fregean thoughts become worldly and a subcategory of Russell's propositions. Bedeutung can then be thought of as playing two roles. First, it assigns to a Russellian property the function which assigns Truth to every individual that has the property and Falsehood to every individual that lacks it. This assignment is a factual, empirical matter, since the property alone does not determine the individuals of which it holds.[12] Second, Bedeutung calculates the values of all complexes, basi-

[11] This difficulty might be avoided by adopting a very strong form of rationalism, of the kind sometimes advocated by Chomsky.

[12] At least not on Russell's metaphysics of the time, which allowed individuals to be 'simples'. Even if individuals were bundles of properties, the property alone would not know which bundles existed.

 

cally by applying functions to arguments, including applying higher order functions to first order functions in ways that needn't concern us here. The calculational role is not empirical, because all the information required for the calculation is contained in the functions on which the calculation is performed. The result of this second role of Bedeutung, the calculational role, is that each definite description will calculate out to an individual, and each sentence will calculate out to truth value.[13]

Fregean thoughts are not, except in exotic cases, about their constituents, they are about the Bedeutung of their constituents. It is odd to say that a sentence is about its truth value, but natural to say that 'The author of *Waverley* wrote *Guy Mannering*' is about the author of *Waverley*, namely, Sir Walter Scott.

Returning now to Russell, we can see how his view of the representational role of language leads directly to the claim that sentences are about what their elements represent (think of 'I met Bertie') and propositions are about their own constituents. So Russell seems to have no need for additional theoretical resources to describe what a proposition is about. However, I see no reason why Russell could not introduce Bedeutung (in both of its roles) explicitly into his semantics. It must appear, at least implicitly, in any calculation of whether a proposition is true and of what a *denoting concept* (see below) denotes.

Like Bedeutung, truth is an *empirical* property of propositions. Now it is natural to try to stay away from the grossly empirical in a semantic theory. Our theory of language should capture features of syntax and semantics that explain the use of language by competent speakers. It needn't tell us which sentences are *true*. That's a job for the special sciences. On the other hand, although semantics need not tell us *which* sentences are true, it should explain what it *is* for a sentence to be true. And for this latter task, the *notion* of Bedeutung is useful. Perhaps we may conclude that semantics should tell us *which* proposition (or *thought*) a given sentence represents (perhaps in terms of *which* constituents the constituents of the sentence represent) and should explain what it is for a proposition (or thought) to be true (perhaps in terms of what it is for the constituents to have a particular Bedeu-

[13] This is a description of Fregean semantics given from a Russellian perspective, one which, for example, takes the notion of worldly properties and relations for granted. If one could take for granted the first Bedeutung relation, the empirical one that holds between the senses of predicates like '_ is red', '_ is hot', '_ is a dog', and '_ is a star' and the functions which assigns Truth to every individual that satisfies the predicate and Falsehood to every individual that does not (the very relation that I called 'fairly mysterious' in the preceding paragraph), Fregean semantics might look much more uniform and elegant. However, there would still, I believe, be the two quite different roles for Bedeutung to play.

 

tung).[14] Understanding what it is for a proposition (or thought) to be true is part of our understanding of what language can be used to do.

Although Russell presents his semantics as Bedeutung-free, the notion does rear its head in one quasi-epistemological corner of Russell's theory, *denoting*.

### *1.1.2 Denoting phrases in* Principles of Mathematics

The 'Denoting' chapter (Ch.5) lays out an exception to the principle that propositions are about their constituents. In the case of certain complex linguistic phrases, in particular but not exclusively, those formed with the six determiners 'all', 'every', 'any', 'a', 'some', and 'the', the corresponding constituent of the proposition is itself to be a complex. But the proposition is not about this complex; it is instead about what the complex *denotes*, an object that is usually *not* a constituent of the proposition, and often not even known to the speaker. I have given one example, here is another: The proposition expressed by 'George IV embarrassed the author of *Waverley*' may be about George IV, *embarrassing*, the novel *Waverley*, and *authorship*, but it is also about Sir Walter Scott, who *is* the author of *Waverley* and the man whom George IV is said to have embarrassed. Scott does not appear to be a constituent of the proposition, and the reporter may not even have known that the man George IV embarrassed was Scott, still the proposition is, in part, about Scott.

Linguists call these phrases *determiner phrases* because of their syntactical structure; they are constructed from *determiners*. Russell called them *denoting phrases* because of their semantical property; they are phrases that denote.[15] Russell called the complexes they express *denoting complexes* or sometimes *denoting concepts* because they are complexes (or concepts) that denote.[16] 'Denoting complex' better conveys what Russell had in mind, but in *PoM* he uniformly used 'denoting concept', so we will follow him in that usage; it is a distinction without a difference. A proposition containing a denoting concept is not about

[14] *How* our semantics tells us which constituent of a Russellian proposition a name represents is a delicate matter. We don't want our semantics to resolve the truth of all identities between names. Russell's own solution, which uses definite descriptions and mixes semantics and epistemology is not ultimately satisfactory.

[15] 'It is plain, to begin with, that a phrase containing one of the above six words always denotes.' *PoM* sect. 58.

[16] In OD, by which time their existence had become dubious, Russell uses *denoting complexes*. Calling them 'concepts' was not, for Russell, a covert way of making them more mentalistic. 'Concept' was used more in the sense of a *classifier*. In *PoM*, all properties and relations are regarded as *concepts* (though not, of course, as *denoting concepts*). Frege also used 'concept' (*Begriff*) in a classificatory, completely non-mentalistic way.

 

the concept but about what the denoting concept denotes. As Russell might have put it, George IV didn't embarrass a *denoting concept,* how would he do *that*; he embarrassed the denotation of the denoting concept, namely, Scott. Though both the linguistic phrase and the concept it expresses are said to denote (and though they presumably denote the same thing), in this chapter of *PoM* Russell seems to focus primarily on the denoting of the propositional constituent, the denoting concept, though it is hard to tell because of Russell's characteristic indifference to the distinction between linguistic expressions and what they express.[17]

> [T]he fact that description is possible—that we are able, by the employment of concepts, to designate a thing which is not a concept—is due to a logical relation between some concepts and some terms [for Russell, 'term' is probably best read as *individual* or possibly *entity*],[18] in virtue of which such concepts inherently and logically *denote* such terms [individuals]. It is this sense of denoting which is here in question. … A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term [individual] connected in a certain peculiar way with the concept. If I say "I met a man," the proposition is not about [the denoting concept] *a man*: this is a concept which does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife. … If we wish to speak of the concept, we have to indicate the fact by italics or inverted commas.[19, 20, 21]

*Denoting* concepts are anomalies, exceptions to the rule and difficult to explain. Yet Russell attached great importance to their role and to the *denoting* relation.

[17] In OD, when denoting concepts have been banished, he returns to denoting *phrases*, and provides an exceedingly 'thin' sense of denoting that applies only to proper definite descriptions (namely, those that succeed in describing exactly one thing).

[18] In *PoM* section 47, Russell writes, 'Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity.' In later developments, he gives slightly conflicting explanations (as readers of Russell would expect).

[19] *PoM* sect. 56. Here, as in all subsequent quotations, bracketed insertions are my comments.

[20] In British English, 'inverted commas' is simply synonymous with what Americans call 'quotation marks'. I have seen reprints of OD in which American editors seem to have struggled to find a special notation for *inverted commas*, especially within the notorious Gray's Elegy passage. In the original, Russell invariably uses double quotation marks except for quotation marks within quotation marks.

[21] Russell here leaves the false impression that the concept *a man* denotes the actual man he met. As we shall see, he explicitly rejects this view. His stereotyping of social classes may also leave a false impression of his views. I am less certain of this.

 

> This notion [*denoting*] lies at the bottom (I think) of all theories of substance, of the subject–predicate logic, and of the opposition between things and ideas, discursive thought and immediate perception. These various developments, in the main, appear to me mistaken, while the fundamental fact itself, out of which they have grown, is hardly ever discussed in its logical purity.[22]

Given all this to *denoting*'s credit, it seems like a lot to sweep away; yet the purpose of OD is, I believe, to sweep away *denoting*. But first, let us look at what progress Russell felt he had made in *PoM* in the analysis of the denoting of his six kinds of denoting phrases. He carefully studied denoting phrases that used 'all', 'every', 'any', 'a', and 'some', distinguishing subtle differences in shades of meaning.[23, 24] He then sets out an apparatus to account for denoting by introducing a new kind of object, conjunctions and disjunctions of individuals (he calls them *combinations of terms*), which will serve as the denotation of certain denoting phrases.

> The combination of concepts as such to form new concepts, of greater complexity than their constituents, is a subject upon which writers on logic have said many things. But the combination of terms [individuals] as such, to form what by analogy may be called complex terms [complex individuals], is a subject upon which logicians, old and new, give us only the scantiest discussion. Nevertheless, the subject is of vital importance to the philosophy of mathematics, since the nature both of number and of the variable turns upon just this point.[25]

He first explains his idea in terms, not of determiner phrases, but sentences with complex grammatical subjects like 'Brown and Jones are courting Miss Smith' and 'Miss Smith will marry Brown or Jones'. He claims that in such contexts, the complex expressions 'Brown and Jones' and 'Brown or Jones' each denote a certain 'combination' of the individuals Brown and Jones: in the first case, a kind of conjunction of them, and in the second case, a kind of disjunction of them. Of the

[22] *PoM* sect. 56.

[23] Interestingly, 'the', which is to figure so centrally in OD is given relatively short shrift in *PoM* (it is discussed in connection with definitions and identity sentences). It is the white sheep of the story; the problem of how to deal with improper definite descriptions (those that do not describe exactly one thing) is not even mentioned.

[24] In some cases, Russell seems to be attempting, in his analysis of the denotation of a denoting phrase, to accomplish what in OD he (and modern logicians) would accomplish through the notion of scope. For example, he argues that 'a point' (as contrasted with 'some point') denotes a *variable* disjunction of points because, '… a point lies between any point and any other point; but it would not be true of any one particular point that it lay between any point and any other point, since there would be many pairs of points between which it did not lie.' [*PoM* sect. 60.]

[25] *PoM* sect. 58.

 

proposition expressed by 'Brown and Jones are courting Miss Smith', presumably a proposition containing the denoting concept expressed by 'Brown and Jones', he says,

> … the proposition is equivalent to, though not (I think) identical with, "Brown is paying court to Miss Smith and Jones is paying court to Miss Smith." … We may call [the kind of conjunction of the individuals Brown and Jones indicated by the "and" in the example sentence] a *propositional* conjunction, since the proposition in which it occurs is equivalent to a conjunction of propositions.[26]

Of the proposition expressed by 'Miss Smith will marry Brown or Jones', presumably a proposition containing the denoting concept expressed by 'Brown or Jones', he says,

> [The kind of disjunction of the individuals Brown and Jones indicated here] is what I shall call the *constant* disjunction, since here either Brown is denoted, or Jones is denoted, but the alternative is undecided. That is to say, our proposition is now equivalent to a disjunction of propositions namely "Miss Smith will marry Brown, or she will marry Jones." She will marry *some* one of the two, and the disjunction denotes a particular one of them though it may denote either particular one.[27]

One might have hoped that Russell would call this the *propositional disjunction* (rather than the *constant disjunction*), thus keeping the notation uniform with his notion of a *propositional conjunction*. However, when at his most creative (which seems to have been most of the time) Russell was free and easy with notational variance. In both examples, there is the view that the propositions expressed by the sentences containing the denoting phrases are distinct from (though equivalent to) the conjunction or disjunction of propositions. This view lies at the heart of the *PoM* theory of denoting. The reversal of this thesis, with the attendant abandonment of *denoting concepts*, lies at the heart of OD.

Russell's *combinations of terms* are much more extensive than that outlined above. They include plurals and, as noted, reflect his attempts to use the nature of the objects denoted to account for what he would later regard as scope phenomena. He was, of course, aware that he was venturing into dubious territory. When he says that denoting concepts all 'denote objects other than themselves', he footnotes the word 'objects' as follows,

> I shall use the word *object* in a wider sense than *term,* to cover both singular and plural, and also cases of ambiguity, such as "a man." The fact

[26] *PoM* sect. 59.

[27] *PoM* sect. 59.

 

> that a word can be framed with a wider meaning than *term* raises grave logical problems.[28, 29]

When he concludes that the five determiner phrases (excluding definite descriptions) 'all men', 'every man', 'any man', 'a man', and 'some man' denote distinct objects, he worries about the objects his theory postulates.

> It appears from the above discussion that, whether there are different ways of denoting or not, the objects denoted by *all men, every man,* etc. are certainly distinct. It seems therefore legitimate to say that the whole difference lies in the objects, and that denoting itself is the same in all cases. There are, however, many difficult problems connected with the subject, especially as regards the nature of the objects denoted. … Consider again the proposition "I met a man." It is quite certain, and is implied by this proposition, that what I met was an unambiguous perfectly definite man: in the technical language which is here adopted, the proposition is expressed by "I met some man." But the actual man whom I met forms no part of the proposition in question, and is not specially denoted by *some man.* Thus the concrete event which happened is not asserted in the proposition. What is asserted is merely that some one, of a class of concrete events took place. The whole human race is involved in my assertion: if any man who ever existed or will exist had not existed or been going to exist, the purport of my proposition would have been different. Or, to put the same point in more intensional language, if I substitute for *man* any of the other class-concepts applicable to the individual whom I had the honour to meet [for example, *student*], my proposition is changed, although the individual in question is just as much denoted as before [i.e. there is just as much reason to think that the actual man is denoted]. What this proves is, that *some man* must not be regarded as actually denoting Smith and actually denoting Brown, and so on: the whole procession of human beings throughout the ages is always relevant to every proposition in which *some man* occurs, and what is denoted is essentially not each separate man, but a kind of combination of all men [presumably, the 'constant disjunction' of all men discussed above].[30, 31]

[28] *PoM* sect. 58.

[29] This is one of my favourite places where Russell carefully notes what may be an insuperable difficulty, and then continues to move ahead. Russell took an admirably experimental attitude toward philosophical theories.

[30] *PoM* sect. 62.

[31] I don't see how changing the proposition by replacing the denoting concept *some man* by *some student* (assuming the man also to be a student) helps to show that the denoting concepts don't denote the actual man. Changing the man himself would show it, if we added the tacit assumption that *some man* has the same denotation in each of its uses.

 

Russell concludes this discussion with a rather sceptical reflection.

> There is, then, a definite something, different in each of the five cases, which must, in a sense, be an object, but is characterized as a set of terms [individuals] combined in a certain way, which something is denoted by *all men, every man, any man, a man* or *some man;* and it is with this very paradoxical object that propositions are concerned in which the corresponding concept is used as denoting. [Underlining added.][32]

The tentativeness of Russell's views about denoting and the theory of propositional functions and variables that he built upon them are quite explicit. In the chapter on propositional functions, he writes,

> The subject is full of difficulties, and the doctrines I intend to advocate are put forward with very little confidence in their truth.[33]

In the chapter on the variable, he writes,

> Thus in addition to propositional functions, the notions of *any* and of denoting are presupposed in the notion of the variable. This theory, which, I admit, is full of difficulties, is the least objectionable that I have been able to imagine.[34]

Worries about what he usually called 'the Contradiction' hover in the background of *PoM*, and are sometimes addressed directly.[35] But they are not the main object of the book.

> The present work has two main objects. One of these, the proof that all pure mathematics deals exclusively with concepts definable in terms of a very small number of fundamental logical concepts, and that all its propositions are deducible from a very small number of fundamental logical principles, is undertaken in Parts II.–VII. of this Volume, and will be established by strict symbolic reasoning in Volume II. … The other object of this work, which occupies Part I., is the explanation of the fundamental concepts which mathematics accepts as indefinable. This is a purely philosophical task, and I cannot flatter myself that I have done more than indicate a vast field of inquiry, and give a sample of the methods by which the inquiry may be conducted.[36]

[32] *PoM* sect. 62. When talking about propositions, Russell seemed to use 'is concerned with' and 'about' synonymously. There is a rather clear example at the end of his 'Descriptions' chapter in *Introduction to Mathematical Philosophy*, hereafter *IMP*.

[33] *PoM* sect. 80.

[34] *PoM* sect. 86.

[35] In the Preface to *PoM*, Russell writes, 'In the case of classes, I must confess, I have failed to conceive any concept fulfilling the conditions requisite for the notion *class*. And the contradiction discussed in chapter ten [titled 'The Contradiction'] proves that something is amiss, but what this is I have hitherto failed to discover.' In Appendix B, which adumbrates the theory of types, he writes on the last page of the book of 'a closely analogous contradiction [concerning the totality of all propositions] which is probably not solvable by this doctrine'.

[36] *PoM* Preface to the first edition.

 

There can be no doubt that 'On Denoting' is a direct attack on the *denoting concepts* of Chapter V of *PoM* and on the 'very paradoxical objects' they were said to denote. It is the central tenet of OD that denoting phrases 'have no meaning in isolation', which is the OD way of saying that there is no propositional constituent corresponding to a denoting phrase (at least none that corresponds in the way that propositional constituents correspond to names, nouns, and adjectives). Here is the very different view of OD:

> *Everything, nothing,* and *something,* are not assumed to have any meaning in isolation, but a meaning is assigned to *every* proposition [sentence] in which they occur. This is the principle of the theory of denoting I [now] wish to advocate: that denoting phrases never have any meaning in themselves, but that every proposition in whose verbal expression they occur has a meaning. The difficulties concerning denoting are, I believe, all the result of a wrong analysis [such as that given in Chapter V of *PoM*] of propositions whose verbal expressions contain denoting phrases.[37, 38]

### *1.1.3 Why did Russell abandon* denoting concepts*?*

I had always assumed that the reason for Russell's change of heart regarding denoting concepts was the difficulty of making the *PoM* theory work, especially for such denoting concepts as *some man*, whose denotation was to be one of those very paradoxical objects, the disjunction of all the men.[39] The alternative was Frege's elegant theory of quantifier phrases—essentially Russell's *everything*, *something*, and *nothing*—as higher order functions on first order functions from individuals to truth values.[40] Frege treats scope as scope. Russell reports that Frege's theory was not known to him when he was writing *PoM*.

> Professor Frege's work, which largely anticipates my own, was for the most part unknown to me when the printing of the present work began; I had seen his *Grundgesetze der Arithmetik*, but, owing to the great difficulty of his symbolism, I had failed to grasp its importance or to understand its contents.

[37] OD p. 480.

[38] Since for Russell, the proposition expressed *is* the meaning of a sentence, the second clause of the principle might be rephrased tautologically as 'every meaningful sentence in which a denoting phrase occurs has a meaning'. Russell's regular use of 'proposition' for both *sentence* and *meaning of a sentence* requires vigilance, but only rarely leads him astray.

[39] I am not suggesting that it *could* not be made to work. Quite the contrary, I think it, or something approximating it, *could* be made to work. For an example, see Parsons (1988). Even the accounting for scope in terms of the object denoted might be made to work, provided we can account for the object denoted in terms of the scope of the denoting phrase, as Russell sometimes seems to do.

[40] Almost, but not quite, Russellian *propositional* functions.

 

> The only method, at so late a stage, of doing justice to his work, was to devote an Appendix to it … If I had become acquainted sooner with the work of Professor Frege, I should have owed a great deal to him, but as it is I arrived independently at many results which he had already established. [41]

Russell had already been careening toward Frege's understanding of quantification in his *PoM* treatment of what he called *formal implication* in the language of logic and mathematics; for this he offered a semantic theory in terms of variables and propositional functions. However, in accordance with the theory of denoting in Chapter V, he argued that the formal implication 'if $x$ is a man then $x$ is mortal' (understood as saying that the corresponding propositional function is true for all values of the variable) expressed a proposition that was distinct from, though equivalent to, that expressed by 'every man is mortal'.

> … consider the proposition [sentence] "any $a$ is a $b$." This is to be interpreted as meaning [i.e. translated into the language of logic and mathematics as] "$x$ is an $a$ implies $x$ is a $b$." [This is Russell's standard formulation of the formal implication, understood as holding for all values of the variable "$x$".] It is plain that, to begin with, the two propositions [sentences] do not *mean* the same thing: for *any* $a$ is a [denoting] concept denoting only $a$'s, whereas in the formal implication $x$ need not be an $a$. But we might, in Mathematics, dispense altogether with "any $a$ is a $b$," and content ourselves with the formal implication: this is, in fact, symbolically the best course.[42]

In sum, the view of *PoM* seems to be that there are two languages, the natural language, which contains denoting phrases, and the much more constrained language of logic and mathematics, which contains open formulas and formal quantifiers. The semantic theory for the former would involve *denoting concepts*; but the semantic theory for the latter can make do with more limited means, perhaps just *propositional functions* and their properties. Many sentences in the denoting phrase language can be 'translated' into sentences of the formal quantifier language. The propositions expressed by such a sentence and its translation will be logically equivalent, but distinct. The grammar of natural language sentences was taken as a guide to the structure of the propositions expressed.

[41] *PoM* Preface to the first edition. Notice the graciously confident counterfactual in the final sentence.

[42] *PoM* sect. 89. I realize that 'every man is mortal' isn't quite of the form 'any $a$ is a $b$'. Russell's obsession with the determiner 'any' is a story I do not fully grasp and have no desire to tell.

 

> On the whole, grammar seems to me to bring us much nearer to a correct logic than the current opinions of philosophers; and in what follows, grammar, though not our master, will yet be taken as our guide.[43]

I had assumed that the abandonment of denoting concepts in OD reflected the fact that when Russell became better acquainted with Frege's theory, he threw in the towel on denoting concepts, and simply used his own variant of Frege's superior theory of quantification.[44] In OD his semantics can be read as if he had tacitly translated the denoting phrase language into the language of logic and mathematics, and then given his semantical analysis for the sentences in *that* language. What Russell claimed to be 'symbolically the best course' for the language of logic and mathematics (by which I assume he meant *the best symbolism* for logic and mathematics) is seen in OD as the best *understanding* of the denoting phrase language. This has the consequence that where in *PoM* we had equivalent but distinct propositions, we now have a single proposition. Translating a sentence of the denoting phrase language into a sentence of the language of logic is no longer seen as yielding a distinct (but equivalent) proposition, but rather as revealing the pre-existing, but hidden, logical form of the denoting phrase sentence.

One of the consequences of the shift is that the burden of establishing the equivalence of the *sentences* of the two languages moves from the science of logic to the art of translation (or *symbolization* as it is now called). This is the affliction that Russell bequeathed to our logic students.

One thing puzzled me. The one denoting phrase whose denotation did *not* seem to require the postulation of a very paradoxical object is the definite description, given short shrift by Russell in *PoM* (though made central by Frege).[45] So why devote 80% of OD to redoing the theory of definite descriptions?[46] The worries about very paradoxical objects in Russell's *PoM* theory of denoting may be good reason to

[43] *PoM* sect. 46.

[44] Russell's variant involves *propositional functions* rather than Frege's *truth valued* functions.

[45] As noted, Russell did not so much as mention the possibility of a definite description being improper in *PoM*, whereas for Frege, the definite description, with its two kinds of 'meaning', became the paradigm of a meaningful expression.

[46] Russell does argue in OD that Frege's sense and denotation theory of descriptions fails to give truth values to certain sentences that should, intuitively, have them. But these arguments seem more of a justificatory afterthought than the real motivation for his drive to rid logic of such expressions. This part of Frege's theory first appears explicitly in 'Über Sinn und Bedeutung'; hereafter S&B. The main ideas are anticipated at the end of section 8 of Frege's *Begriffsschrift* (1879). Frege's article is translated and reprinted almost everywhere that OD appears. Beware versions of S&B in which the title is translated as 'Sense and Meaning', lest you fall into Russell's misunderstandings of Frege.

 

favour Frege's treatment of the *quantifier* determiner phrases 'all men', 'every man', 'any man', 'a man', and 'some man', but *those* reasons did not seem to argue for a similar recasting of the semantics of definite descriptions. Indeed, Frege showed the way by *not* treating them similarly, and Russell knew it.[47] Furthermore, it couldn't be, as Strawson (1950) would insist, that Russell was motivated by a concern to find a treatment of definite descriptions that ensured that sentences containing improper descriptions remained meaningful. Russell's treatment of definite descriptions in *PoM already* gave meaning, even a *meaning in isolation*, to all definite descriptions, proper as well as improper. So I concluded that when Russell started 'eliminating' denoting phrases (by implicitly translating into the language of logic), he just got carried away.

I was wrong.

### *1.2 The Contradiction*

#### ***1.2.1 Urquhart's Discovery***

In his illuminating 'Introduction' to Russell's papers in logic during the period 1903 to 1905,[48] Alasdair Urquhart uses Russell's correspondence during the period between *PoM* and OD to demonstrate that the goal of the development of the theory of descriptions in OD was to find a way around 'the Contradiction'. As Urquhart writes,

> Most of the very voluminous secondary literature on Russell's Theory of Descriptions discusses it in isolation from its setting in the enterprise of the logical derivation of mathematics; the resulting separation of the logical and mathematical aspects of denoting is foreign to Russell's own approach.[49]

It is a simple historical fact that Russell's work on denoting was done *in the course of* his attempts to solve the contradiction. But we now know that Russell himself saw his work on denoting as *in aid of* that project.

Here is an eye-opening passage from an April 14, 1904 unpublished letter unearthed by Urquhart:

> Alfred [North Whitehead] and I had a happy hour yesterday, when we thought the present King of France had solved the Contradiction; but it turned out finally that the royal intellect was not quite up to that standard.[50]

[47] See the discussion of Frege in OD on p. 483.

[48] 'Introduction' to *CP4*; hereafter 'Introduction.'

[49] 'Introduction' p. xxxii.

[50] From a letter to Alys Pearsall Smith, Russell's then wife, quoted in 'Introduction' p. xxxiii.

 

This unmistakably connects the problem of how to treat improper definite descriptions with the Contradiction.

In a previously published retrospective letter of March 15 1906, Russell wrote,

> In April 1904 I began working at the Contradiction again, and continued at it, with few intermissions, till January 1905. I was throughout much occupied by the question of Denoting, which I thought was probably relevant, as it proved to be. … The first thing I discovered in 1904 was that the variable denoting function is to be deduced from the variable propositional function, and is not to be taken as an indefinable. I tried to do without ι as an indefinable, but failed; my success later, in the article 'On Denoting', was the source of all my subsequent progress.[51]

What Russell 'discovered' seems to be that first, a singular denoting phrase like '*x*'s father', which Russell may have thought of as standing for a function from individuals to individuals (a *denoting function)*, can be put into a standard form by introducing the formula '*y* fathered *x*',[52] which may be thought of as standing for a *propositional* function, and then using the iota operator, which picks out the unique argument to the propositional function that yields a true proposition. This allows us to form a definite description that can replace the original denoting phrase. So the singular term '*x*'s father' can be put into the standard form of a definite description, 'the *y* such that *y* fathered *x*'. This does not rid us of singular denoting phrases, but at least we have put them all in one form.

Perhaps it was this consolidation and focus on uniqueness that brought Russell to the critical insight for the second step, which we may put as follows: that definite descriptions are nothing more than indefinite descriptions with uniqueness added. Thus, the definite descriptions 'the *y* such that *y* fathered *x*' can be transformed into the *in*definite description 'a *y* such that (*y* fathered *x* and only *y* fathered *x*)'. Russell was already translating sentences containing indefinite descriptions into a formalism using existential quantification. So the so-called *contextual elimination* of definite descriptions can be seen as reducing to two steps, first the replacement of the definite description by an indefinite description with uniqueness added, and second, the contextual elimination of sentences containing the indefinite descriptions in favour of what amounted to existential generalizations. The second step

[51] Russell always insisted on writing this in biblical form as '*y* begat *x*'.

[52] From a letter to Philip Jourdain, quoted in 'Introduction' p. xxxiii. But note that 'all my subsequent progress' here refers to only a nine month period.

 

is something Russell took for granted when working in the language of logic.[53, 54]

In this way, all singular denoting phrases, i.e., complex singular expressions, are ultimately eliminated from the language.[55] The interesting question at this point is why did Russell think this a worthy goal, and why did he think it a help in resolving, or avoiding, the Contradiction? Most authors think that it was the theory of types that avoided the Contradiction and that the elimination of singular denoting phrases was irrelevant.

It may be that part of the importance that Russell attached to his theory of descriptions was really due to the liberating effect of what he called his *principle of denoting*: that although denoting phrases have no meaning in isolation, we can systematically explain the meaning of every sentence in which they occur. Russell came to call expressions to which this principle applies 'incomplete symbols'.[56] A significant use of this idea occurs in *PM* where the expression for the *extension of a propositional function* is treated as an incomplete symbol.[57] However, the treatment of *extensions of propositional functions* is fundamentally different from that of definite descriptions. Whereas the theory of descriptions analyzes the use of definite descriptions in terms of whether or

[53] This is not the way Russell usually puts it, though he comes close to this formulation in the first paragraph of his discussion of 'the' in OD. This very clever idea may suffice for Russell's purposes, to found mathematics on logic, but probably does not work in general, for example in the case of 'Some Greeks worshipped the sun-god'. (This case is drawn from an example of Alonzo Church's and from Russell's analysis of 'Apollo' as abbreviating a definite description.) Note that the problem does not lie in the treatment of definite descriptions as indefinite descriptions with uniqueness added (which does seem to work), but rather in the second step, the treatment of indefinite descriptions as interpretable by existential generalizations.

[54] Urquhart reports in his 'Introduction' that 'Peano had already suggested the device of contextual definition in a monograph on mathematical logic read by Russell'. Although Peano's operator selects the unique element of a class rather than the unique thing satisfying a description, the connection with Russell's contextual definition of definite descriptions is obvious.

[55] I recognize that I am speaking somewhat loosely about contextual eliminations. But the amount of apparatus required to be precise, especially about scope in natural language, is more than the purposes of this article can carry. Russell's efforts in this direction amount to his saying: 'I use "C($x$)" to mean a proposition in which $x$ is a constituent' and then telling us in a footnote that 'C($x$)' really means a propositional function. It seems that 'C' is for *context*, but the switch between linguistic context, when he writes things like 'C(no men)', and the propositional function expressed by 'C($x$)', when he writes things like 'C($x$) is always true', beclouds his exposition.

[56] See Ch. 3, 'Incomplete Symbols', of the Introduction to *Principia Mathematica*; hereafter *PM*.

[57] Propositional functions are *intensional* in exactly the following sense: two propositional functions may assign true propositions to the same individuals while remaining distinct. The *extensions* of such propositional functions should be such that if distinct propositional functions assign true propositions to the same individuals the extensions of the two functions will be identical. A natural way to think of the extension of a propositional function F is as a function that assigns to

 

not there exists a unique individual so described, *PM* analyzes sentences containing expressions for the extension of a propositional function in a way that makes it *irrelevant* whether or not there exist such things. The *PM* theory is a theory of *virtual* extensions; we have the singular denoting *phrases* that purport to denote such things, but the talk of such things is explained away as talk of other kinds of things, so intuitively, they *never* denote.[58] Thus the treatment of extensions in *PM* is not based on the treatment of definite descriptions, that is, it does not 'reduce' such expressions to definite descriptions. The treatment of extensions is based rather on the principle that if an explanation can be given for every sentential context in which a given expression appears, no further assignment of meaning to the expression itself is required.[59, 60] One way of assigning meaning to every sentential context is to start by assigning a meaning to the given expression, but Russell's development of his theory of descriptions showed him that there were other ways.

I think it not unlikely that it is the liberating effect of his *principle of denoting*—the opportunity to sweep away troublesome entities in favour of the *virtual*—that Russell had in mind in attributing 'all my subsequent progress' on the Contradiction to his theory of descriptions. But I do not think the *PM* treatment of extensions of propositional functions is important from a logical point of view,[61] so I

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each individual $x$ the truth value of the proposition F($x$). Instead of truth values, any fixed pair of a true and false proposition would suffice. The extension of a propositional function can also be thought of as the characteristic function of a class (the class of individuals to which the extension of the function assigns Truth (or the fixed true proposition). Russell seems to have thought of extensions of propositional functions this way, calling them 'classes'. It is thus, in replacing extensions of propositional functions with virtual extensions, that Russell's 'No Class' theory comes about.

[58] I say 'intuitively' because we don't have a precise definition of *denoting* in the case of extensions in *PM* like the one Russell gives us in OD for definite descriptions. Note that the definition in OD does correspond to the intuitive notion.

[59] This method promised to be much more useful than I think it has turned out to be in the hands of those philosophers who have used it. It is a natural treatment for only a few singular expressions, such as 'the average sophomore' in 'The average sophomore enrolls in 4.2 classes and completes 3.8 of them'. One may hesitate to use Russell's theory of descriptions in such a case since it requires the hypostatization of an abstract sophomore. It seems more natural to understand all contexts of 'the average sophomore' in terms of statistical claims about the actual sophomores. However, in my view, it is usually better to hypostatize, perhaps even in a case like this, and certainly in the treatment of extensions of propositional functions.

[60] Gödel (1944) expresses doubt that Russell *has* given a meaning to all such sentential contexts because the syntax of *PM* is so ill-explained that it is impossible to tell what all the sentential contexts *are*.

[61] It may have been historically important to philosophers who sought to ape its method.

 

continue to look for more interesting ways in which the theory of descriptions might have been seen as relevant to the Contradiction.

In the following I will show how the problem regarding the Contradiction in Frege's *Grundgesetze* first appears as a problem about *denoting*. This would make it natural for Russell to think that a new theory of denoting could be helpful in clarifying that part of the problem.

It takes some time to tell the story, but it is an interesting and important story, worth the telling in its own right. However, much of it is tangential to OD. So the impatient reader can take my word for it, and jump directly to Part 2 on page 968.

### *1.2.2 The incompleteness of linguistic expressions and functions*

The language of mathematics is largely a language of operation symbols (i.e. functional expressions, like '+' and '÷') rather than predicates, and Frege's logic was well-suited to it.[62] The language freely allowed functions from entities of every kind to objects. Among the primitive signs of the language is an operator which, when applied to a functional expression, yields the name of the course-of-values of that function.[63]

In Frege's metaphysics there is a fundamental divide between functions (which are incomplete or *unsaturated* things) and objects (which are complete or *saturated* things). The distinction seems to derive from Frege's syntactic view that a sentence like 'Dion walks' should be parsed into components of which one contains the gap resulting from the *literal* removal of the other from the whole. Thus the parts may be 'Dion' and '_ walks' (or perhaps 'walks' and 'Dion _'). In the case of true functional expressions, for example '$(2 + 3x^2)x$' Frege was especially concerned to isolate the function name from the argument expression.

> The essence of the function manifests itself … in the connection it establishes between the numbers whose signs we put for "$x$" and the numbers that then appear as denotations of our expression … Accordingly the essence of the *function* lies in that part of the expression which is over and above the "$x$". The expression for a *function* is *in need of completion*, *unsaturated*.[64]

[62] I refer here to the ill-fated system of *Grundgesetze*. The introductory sections, plus the Appendix to Volume II in which Frege discusses the Contradiction, are translated by Montgomery Furth as *The Basic Laws of Arithmetic*; hereafter *Basic Laws*. All translations are taken from *Basic Laws*.

[63] *Course-of-values* is Furth's translation in *Basic Laws* of Frege's 'Werthverlauf'. It is unrelated to so-called course-of-values induction. Frege's term is also often translated as 'value range' or 'range-of-values'. When the function is a concept (i.e. a function to the two truth values) it is also referred to as the 'extension of a concept'; Frege talks this way, but it is dangerous talk.

[64] Section 1 of *Grundgesetze*, emphasis in the original.

 

We see the characteristic Fregean elegance in this coordination of linguistic incompleteness with the incompleteness of a semantically associated entity.

But is Frege's argument for linguistic incompleteness plausible? He seems to be proposing a syntactic theory according to which compounds are built from 'incomplete' expressions. However, syntactic theory doesn't work like that. The syntactic operations (functions) that yield compounds from components needn't do so by filling gaps. Furthermore not every literal removal of a part yields a function that is implicit in the parsing. So for example, in 'Bertie met the father of Charles IV' the function 'Bertie met the father of _' does not represent a stage in the parsing of the former. Perhaps the syntactic functions that yield compound expressions when applied to their parts are unsaturated (though I see no reason why they need be), but there is certainly no need for incomplete *expressions*. Basically, *nothing* (that is, no well-formed part of language) remains when we remove 'Charles IV' from 'Bertie met the father of Charles IV', nothing more than what remains when we remove the 'cat' from 'cattle' (to cite a well-known example of Quine's). We can, of course, make substitutions on component expressions at any level, but it isn't a matter of gap filling. When we substitute 'bad' for 'good' in 'Bertie made the best choice' we get 'Bertie made the worst choice'. Where's the gap? Frege's incomplete expressions, formed by extraction, seem to be of his own creation. There *are* ways of building well formed expression that mimic Frege's operation. We build the 'complete' expression by using a variable, and then adding an operator. Instead of the incomplete '$((2+3\_^{2})\_)$', we construct '$\lambda x\,((2+3x^{2})x)$'. As we shall see shortly, Frege is aware of such operations, but the ideology of incomplete expressions yielding incomplete entities and complete expressions yielding complete entities is unshakeable. So Frege cannot allow our gapless '$\lambda x\,((2+3x^{2})x)$' to stand for a function.

There is also in Frege's syntactical discussions, and that of his commentators, more than a whiff of the problem of forming unities from pluralities, how do we obtain a single entity, a sentence, say, from a plurality of words? What makes the string of words a *sentence* rather than just a list of words, as they would be (in English), if written vertically rather than horizontally? Is it the yearning of the incomplete predicate that is the glue that holds the parts together to form a single sentence?[65] And is it the yearning of the function itself that allows the function to

[65] On unsaturatedness as glue, 'For not all the parts of a thought can be complete; at least one must be "unsaturated," or predicative; otherwise they would not hold together.' From 'On Concept and Object' p. 54 in *Translations from Frege*.

 

metabolize its argument and form a value (rather than having the argument just sit there, like a lump, inside the function). In reply, I would say that we need to think in terms of an algebra of expressions, not the literal pushing together of tokens. (How close together must the words get to form a sentence? A millimetre? An inch? A foot? A yard?) Although linguistic compounds often display their parts (for better readability), they don't always, as for example in 'better', which has 'good' as a part. The claim that every language must contain a notation for application of function to argument (or some other syncategorematic expression that includes a notion for application of function to argument) derives from this point of view. But every logician familiar with Polish notation knows this to be false. (Note that juxtaposition is not a symbol.)

In sum, (1) we don't need incomplete expressions to account for the unity of compound expressions, (2) we don't need incomplete expressions to account for the syntactical structure of compound expressions, (3) even if we counted the formation of compound expressions as gap filling, we would still not need (or want) *arbitrary* incomplete expressions formed by literally extracting any part from a compound expression (because not every extraction corresponds to a parsing), and (4) to form an expression for a function from an expression for one of the values of the function, we proceed not by way of extraction but by way of addition (of variables and an operator).

Hence, there is no foundation for the theory of unsaturated functions in a syntactical theory of incomplete expressions.[66]

### *1.2.3 Functions objectified*

According to Frege, it is these incomplete expressions that denote functions, which, in homage to Frege's grammar, are likewise incomplete. However, each incomplete function has a corresponding course-of-values, which is a complete thing, an object. In exact analogy, each incomplete expression can be completed by filling its gap with a variable and prefixing a variable binding operator. For example, from the incomplete expression '$((2+3\_^{2})\_)$' we obtain the complete '$\lambda x\ ((2+3x^{2})x)$'. Let us call such a complete expression a *course-of-values abstract.*

[66] Frege's distinction is, I believe, founded on an error, the error of thinking that syntax requires the notion of an *incomplete symbol.* (I argued that this is an error in my 1964 Dissertation *Foundations of Intensional Logic.*) This error, I believe, may stem from a more deep-seated error, the error of thinking that any unity with parts must have an incomplete part to lock the other parts together. Regarding functions, it is in the nature of a function that *yields* a value when *applied* to an element of its domain. If those metaphors of functional activity (as compared to the inertness of mere correlations) make functions 'unsaturated', so be it.

 

Many suggestions have been made as to what these 'complete' objects *are*. But the most natural thing to think is that the course-of-values of a function simply *is* the function, conceived of *qua* object (like the Evening Star seen in the morning). There is no way to say, within Frege's language, whether this is right or wrong, because the identity claim would be ill-formed.

Frege held that there are two distinguished, truth-value like objects, which he called *The True* and *The False*.[67] An ordinary predicate, like '_ is human', which I represent as Frege would as an *incomplete symbol*, was held to denote a function from objects to these distinguished objects. Frege called functions to the two truth values, *concepts*.[68] Since functions are incomplete entities, we have the desired result (for Frege) that an incomplete expression denotes an incomplete entity.[69]

Such a function might be thought of, in modern day terms, as the characteristic function of a class (the class of objects to which the function assigns *The True*). But to Frege, and to Russell at the time of *PoM*, the function could not *be* the class because (and only because) classes were objects. So they took the courses-of-values of such functions (courses-of-values already being objects) to be classes.[70] It is not clear why this happened. My best guess is that somehow both Frege and Russell, encouraged by confused talk about courses-of-values being the

[67] In *Grundgesetze,* these objects are identified with particular courses-of-values.

[68] We translate Frege's term for such functions ('Begriff') as 'concept', but again, as with Russell's *denoting concepts*, there is nothing cognitive about them, and, in Frege's case, there is nothing even intensional about them. As noted, both Russell and Frege used 'concept' in the sense of 'classifier'. See n. 16.

[69] I repeat, there is nothing cognitive or intensional about Frege's *incomplete* functions. What differentiates them from the current, primitive notion of a function (primitive, as compared with a representation, for example as a set of ordered couples) is that they are one of a pair of entities in Frege's metaphysics that are fused in the current notion. Though even Frege calls courses-of-values *extensions* of concepts, all of Frege's functions are themselves as extensional as can be. These functions, incomplete though they are, are the denotation, the *Bedeutung*, of predicates, not the sense. Frege's complete/incomplete distinction is entirely his own and is entirely independent of the distinction between sense and denotation (which also shows up in Russell's distinction between the meaning and the denotation of a denoting phrase). As noted, the former is 'syntactic', both at the level of expressions and of the entities they denote. In both cases the concern is with parts fitting together to form wholes (unity from plurality). The sense/denotation distinction is semantic; it concerns how denotation is determined. The extensionality of Fregean functions is beautifully argued in Montgomery Furth's valuable 'Editor's Introduction' to *Basic Laws*. See especially sect. 6 pp. xxxvii–xlviii. *Russellian* propositional functions are not extensional. They do have plausible extensions.

[70] Among the minor reasons to question this identification is that although a class may be determined by its characteristic function, the function is not determined solely by the class. The characteristic function is also dependent on what there is outside of the class. In a normal set theory, characteristic functions don't exist because their domains would be the universal set.

 

*extensions* of such functions, talk that Frege consistently indulges in, precipitously identified the courses-of-values with an already known entity, the class.[71] Of course, as noted, Frege's unsaturated functions are already as extensional as can be.[72] In fact, they (or a surrogate that uses two distinguished propositions, say, a particular tautology and its negation, to replace Frege's truth values) are already the natural entities to serve as the extension for one of Russell's equally unsaturated, but intensional, propositional functions. This suggests a rather natural treatment of extensions of propositional functions in Russell's type theory, one that uses these surrogates for Fregean concepts and does not depend on contextual elimination. But that's another story.

[71] In reading the following, keep in mind that for Frege, a *concept*, though unsaturated, is simply an extensional function from individuals to the two truth values. Russell writes to Frege on July 24, 1902, 'every day I understand less and less what is really meant by "extension of a concept"'. Frege responds on August 3, 1902, 'You find doubtful whether concepts with the same extension have the same range of values. Since for me the extension of a concept or class is only a special case of a range of values, concepts always have the same range of values if they have the same extension; for the extension is the range of values.' Russell replies on August 8, 'many thanks for your explanations concerning ranges of values. … But I still lack a direct intuition, a direct insight into what you call a range of values …'. On September 23, in an effort to resolve the Contradiction by depriving the dangerous concepts of their courses of values, Frege writes, 'it may be asked whether there is not a characteristic mark by which those … concepts that have a class pertaining to them can be distinguished from those that have no extension; and here I am using the word "class" for "extension of a concept"'. The letters all appear in Frege's *Correspondence*. It should be added that Frege was severely critical of the theory of sets ('Mengen'), explicitly on grounds of vagueness, but perhaps because he did not see how to reduce that theory to logic. But he advocated a theory of classes ('Klassen'), which he *defined* as the extensions of concepts. This he regarded as already a logical notion.

[72] Despite Frege's seeming disclaimer that '$(x)(f(x) = g(x))$' is as close as one can get to saying that the function $f$ is identical with the function $g$, I do not think that this is the case. In particular, since Frege has second order functions that carry first order functions to objects, we should be able to adopt Leibniz's idea of using indiscriminability to define identity. The fact that Frege does use exactly this method to define identity between objects shows that he believed that there were enough functions to discriminate between every non-identical pair. The identity of the first order functions $f$ and $g$ would be expressed, using a second order variable '$M$', by '$(M)(M(f) = M(g))$'. Then the extensionality of functions should be expressed by,

$$(f)(g)[(x)(f(x) = g(x)) \supset (M)(M(f)=M(g))].$$

Let us call this the *Principle of Extensionality* for functions. In Frege's notation, an additional operator is required after the '$(M)$' to fill the hidden argument place of the final '$f$' and '$g$', so the foregoing isn't quite well-formed in his notation, but a well-formed version can be formulated. Given what Frege says about his intentions, this *Principle of Extensionality* should be a theorem of his system. At the moment I don't know its standing in *Grundgesetze*, but I trust that others do. What I have called 'Frege's seeming disclaimer' is my reading of the following passage quoted by Furth on p. xliv of *Basic Laws* from Frege's review of Husserl: '… coinciding in extension is a necessary and sufficient criterion for the holding between concepts of the relation corresponding to identity for objects. (Identity does not in fact, properly speaking, hold for concepts.)' Where concepts are involved, Frege's talk of 'coinciding in extension' is equivalent by his Axiom V (see below) to my '$(x)(f(x) = g(x))$'. I have avoided talking about extensions or other courses-of-value because it is the *functions* whose extensionality we want to establish, and because courses-of-values are already problematic.

 

I think that we should ignore this identification of the course-of-values of a concept with a class, and just think of courses-of-values as mysterious objectifications of functions. Mysterious, because I have trouble keeping a fix on why Fregean unsaturatedness makes functions unfit for duty as numbers.[73] Frege has no compunction about quantifying over functions. So the requirement that numbers be saturated (and thus *objects*) is presented in a way that seems to flow from the metaphysics rather than from the expressive or derivational demands of the logic. The situation would be quite different if Frege had said, 'When I proposed that the natural numbers were functions, I discovered that I could not prove that there were a sufficient number of these functions. Then I realized that if there were an object at the bottom of the hierarchy corresponding to each function, I could succeed in the proof'. This, in essence, is what Russell and Whitehead said about the Axiom of Reducibility. In this situation, there would have been no elaborate discussion of incomplete expressions. So the introduction of courses-of-values seems *motivated* by metaphysics; they are certainly given an elaborate metaphysical justification.[74,75] This is not to deny their importance in satisfying the derivational demands of Frege's project (though they do so at a heavy price). Russell, in *PoM,* accepts something like Frege's metaphysical distinction, with the courses-of-values of a propositional function taken to be what Russell called a 'class as one'. In section 104 he writes, 'A class as one, we shall say, is an object of the same *type* as its terms; i.e. any propositional function $\Phi x$ which is significant when one of the terms is substituted for $x$ is also significant when the class as one is substituted.' [76]

### *1.2.4 The notorious Axiom V*

Frege wrote an axiom, the notorious Axiom V, which embodies the idea that courses-of-values are merely objectifications of functions. The axiom states the principle of individuation for courses-of-values:

[73] Frege famously insisted that numbers had to be objects (for the usual syntactical reasons). In the end, Russell made them functions.

[74] A justification that I find completely unpersuasive.

[75] I do not and would not argue that Frege's distinction may not be suggestive of various *other* important and valuable distinctions, the class versus set distinction broached in von Neumann's (1925) set theory immediately springs to mind. But what it *is* and what it *suggests* are two different things.

[76] In saying that a class as one is 'an object of the same *type* as its terms', Russell means such a class of individuals counts as another individual from a type theoretical point of view. Thus, if a propositional function $\Phi(x)$, from individuals to propositions, has a class as one associated with it, that class is among the values of '$x$'.

 

courses-of-values are identical if and only if they are the courses-of-values of identical functions (functions with the same domain that assign the same value to the same argument). Given that courses-of-values are just the objectification of functions, how could it be otherwise?

Let us use the notation '$\lambda x f(x)$' for the course-of-values of the function $f(\xi)$, (where '$\xi$' is a gap holder).[77] Then we can write Axiom V as follows:

Axiom V: $(x)(f(x) = h(x)) \equiv \lambda x f(x) = \lambda x h(x)$

Note that Axiom V does not appear to assert that certain, or all, functions *have* courses-of-values. That claim is embedded in the syntax and logical rules of Frege's language, in particular, in the rules that treat course-of-values abstracts like any other (denoting) singular expression and allow existential generalization, using an *object* variable, upon them.[78] The Axiom simply states the obvious principle of individuation for courses-of-values: same function, same course-of-values; different function, different course-of-values.

The axiom can be given a seemingly innocent justification. Let's call the left to right direction of Axiom V, 'Axiom Va' and the right to left direction of Axiom V, 'Axiom Vb'. Axiom Va is a simple principle of extensionality for course-of-values. It is, or should be, derivable from the other axioms.[79] I'll return to this.

Now what about Axiom Vb? Suppose, in an effort to understand what these mysterious courses-of-values *are*, we propose to investigate the relation that holds between the objects that satisfy the function and the function's course-of-values. Let's call this relation 'R'.

Def: $\quad R(x, \lambda y f(y)) \equiv f(x)$[80]

What can we learn about this relation? By Leibniz's Law, we get:

[77] This is not written in Frege's actual symbolism, but it tracks Frege's symbolism. I write '$\lambda x$' for word processing convenience and for familiarity, where Frege wrote '$\grave{x}$', a variable with a 'smooth breathing' diacritical mark above it. Frege also has conventions concerning the use of Latin, Greek, and Gothic letters, which I ignore. Frege's gap-holder notation is an improvement over the gappy notation because it can identify which gaps are to be filled by the same thing and which may be filled by different things.

[78] In *Grundgesetze*, '$(f)(\exists y) y = \lambda x f x$' is a theorem, derivable from '$(f)\lambda x f x = \lambda x f x$', which is derivable from '$(x)\, x = x$'. So if we wish to deny that some particular function F($\xi$) *has* a course of values, we cannot do so by writing, '$\sim\exists x\, y = \lambda x F x$'. That would only produce inconsistency (of which no more is needed).

[79] Axiom Va follows immediately from my second order *Principle of Extensionality* in n. 72 (which may or may not already be a theorem in *Grundgesetze*, but should be).

[80] Frege likes relations to be defined 'everywhere', so let's stipulate that if $y$ is not a course-of-values, '$R(x,y)$' is false.

 

$$\lambda x f(x) = \lambda x h(x) \supset \mathrm{R}(x, \lambda y f(y)) \equiv \mathrm{R}(x, \lambda y h(y))$$

From this, using Def, we can derive:

$$\lambda x f(x) = \lambda x h(x) \supset f(x) \equiv h(x)$$

and thus,

$$\lambda x f(x) = \lambda x h(x) \supset (x)(f(x) \equiv h(x))$$

which is Axiom Vb. Interesting![81]

If we thought of courses-of-values as sets, we might think of the relation we call 'R' as that of set membership, $\in$. So let's try that.

(E): $\quad (x \in \lambda y f(y)) \equiv f(x)$

Since (E) has the form of Def, we could take '$\in$' as a primitive, and by repeating the argument given in connection with 'R' establish that (E) implies Axiom Vb. Also, with '$\in$' as a primitive, we could define course-of-values abstracts $\lambda x \Phi x =_{\mathrm{DF}}$ The $y$ $(x)(x \in y \equiv \Phi x)$. Using Russell's OD theory of descriptions, (E) and Axiom Va imply that all of the abstracts are 'proper'.

Now in *Grundgesetze* '$\in$' is defined in terms of course-of-values abstraction:

$$x \in y =_{\mathrm{DF}} (\exists f)(y = \lambda z f(z) \,.\, f(x))$$

Using this definition, we can immediately prove that Axiom Vb implies (E).[82]

So, to sum up, we can take course-of-values abstracts as primitive, define '$\in$', and Axiom Vb will be equivalent to (E). Or, we can take '$\in$' as primitive, define course-of-values abstracts, and Axiom Vb will be equivalent to (E). So (E) holds a key to the dual nature of Axiom Vb. If (E) tells us that '$x \in \lambda y f(y)$' says nothing more than '$f(x)$', then it tells us that $\lambda y f(y)$ is nothing more than the objectification of the function $f(\xi)$. On the other hand, if (E) tells us that for any function $f(\xi)$, $(\exists y)(x)(x \in y \equiv f(x))$, then it tells us that there are sets.

Once we have (E) we can verify our earlier claim that Axiom Va is the analog for courses-of-values to the Axiom of Extensionality for sets. Using Axiom Va and (E) we can immediately prove:

[81] Innocent though it seems, it is Axiom Vb that brings inconsistency to *Grundgesetze*. So it couldn't *really* be derivable from a proper definition. I'll leave the further study of how seeming innocence can mask evil as an exercise in the virtues of logic.

[82] By immediately *prove*, I mean that each step is secure and does not veer into the neighbourhood of the Contradiction.

 

(Ext): $(y)(y \in \lambda x f(x) \equiv y \in \lambda x h(x)) \equiv \lambda x f(x) = \lambda x h(x)$

So Axiom V is, in part, an analogue to the traditional axiom of extensionality, which gives the identity conditions for sets; it give the identity conditions for courses-of-values. But in saying that courses-of-values are as fine-grained as their functions, the axiom shows its dark side.

We now know, from Cantor, that not all functions from objects to truth values can be objectified. Russell, who had been reading Cantor, was thus lead to his formal derivation of the Contradiction in Frege's system.[83] Both Frege and Russell quickly put the blame on Axiom V.[84] It is obvious that without Axiom V, Russell's proof of the Contradiction wouldn't go through,[85] because without Axiom V, we might take Julius Caesar to be the common course-of-values for every function. But then courses-of-values would not be what they were intended to be, *objectifications* of functions.

With or without (E), in Frege's logic every course-of-values abstract must denote *something*. So when it is said that we can deny that certain functions have a course-of-values by meddling with Axiom V, what is meant is that we can identify the course-of-values of a particular function with something that it isn't, and thus rob it of its natural powers. To proceed in this way, is to think that Russell's robust sense of reality would allow him to deny that Hamlet exists by identifying Hamlet with Shakespeare. But this would be doubly wrong in that it doesn't make 'Hamlet exists' false and it does make 'Hamlet = Shakespeare' true. Those two wrongs don't make a right. In Frege's logic we must distinguish the singular term *denoting* (all singular terms denote) from its

[83] Russell acknowledges the debt to Cantor in *PoM* section 100. The introduction to the first volume of *Grundgesetze* contains a poignant example of the truism: Be careful what you wish for! Frege writes, with bravado, about how he hopes his work will be received, 'Not that only a laudatory review could satisfy me; on the contrary, I should far prefer an attack that is thoroughly well informed than a commendation in general terms not touching the root of the matter'. In the introduction to the *second* volume, he writes, 'Hardly anything more unwelcome can befall a scientific writer than that one of the foundations of his edifice be shaken after the work is finished'.

[84] Frege himself, both in his Introduction to *Grundgesetze* and in the Appendix to the second volume (1903) in which he discusses Russell's derivation of the Contradiction, casts doubt on Axiom V and writes as if it asserted the existence of the course-of-values of a function. 'It is a matter of my Basic Law (V). I have never concealed from myself its lack of the self-evidence which the others posses, and which must properly be demanded of a law of logic, and in fact I pointed out this weakness in the Introduction to the first volume. … Is it always permissible to speak of the extension of a concept, of a class?' (Appendix to the second volume.) One might think that a dose of Russell's theory of descriptions would be salutary right here, in that it shows how while maintaining bivalence, we can 'speak of', i.e., introduce a notation for, without assuming that the notation is 'proper'. In the preceding, it is (E) that makes this claim.

[85] Nor would most of Frege's derivations of theorems that are central to the logicist program.

 

being *proper* (denoting what it professes to denote). In Russell's theory of descriptions these two are identified.

### *1.2.5 The ontological commitments of syntax*

Two of Frege's ontological ideas are enshrined in the syntax of *Grundgesetze*. The first is that the functions form a hierarchy separate from the objects. Frege believed this hierarchy to be 'deep in the nature of things'. He even thought that the relation between an object and a first-level function was a *different relation* than that between a first-level function and a second-level function.[86] These fundamental ontological differences, their belonging to different realms, allow the functions of different levels to have distinct domains. And it is only this that justifies the syntactical restrictions that institute Frege's type theory, because every function must be applicable to everything in its domain. It is fundamental to Frege's view of logic that *every complete expression must denote*. Frege saw this as requiring that there be no partial functions. A function's domain is always a complete realm. So he requires that the *plus function* assign a denotation to 'the sun + 1', though this could be done arbitrarily.[87] Thus, it is only because the first-level function $\xi \notin \xi$ belongs to a different realm from its arguments, that Frege justifies the type theoretic syntax that rules out application of the function to itself as being ill-formed. (And it is only because the course-of-values of this function does belongs to the realm of the function's arguments that application of the function to its own course-of-values *is* well-formed.)

As is well-known, in Frege's theory of sense and denotation, it is not expected that every singular term will have a denotation. Sentences containing singular term that do not denote will lack a truth value.[88] But Frege believed that *logic* required every well-formed expression to denote. One may speculate that this view was derived from the expecta-

[86] '[the difference between first level and second level functions] is not made arbitrarily, but founded deep in the nature of things.' From 'Function and Concept' (1891) in Gottlob Frege *Translations from the Philosophical Writings of Gottlob Frege*; hereafter *Translations from Frege*. In the following, remember that a *concept* is, for Frege, just a function whose value is always one of the two truth values. 'Second-level concepts, which concepts fall under, are essentially different from first-level concepts, which objects fall under. The relation of an object to a first-level concept that it falls under is different from the (admittedly similar) relation of a first-level to a second-level concept. (To do justice at once to the distinction and to the similarity, we might perhaps say: An object falls *under* a first level concept second; a concept falls *within* a second level concept.)' From 'Concept and Object' in *Translations from Frege*.

[87] 'What rules we lay down is a matter of comparative indifference.' From 'Function and Concept'.

[88] Because Frege calculates truth values by applying functions to arguments, it is likely that he thought that if, at any point, there were no argument to which to apply the relevant function, no value for that function would ensue. Thus, in the end, no truth value would ensue.

 

tion that a non-bivalent logic would be impossibly cumbersome. Russell's criticisms of Frege in OD, wherein he states that 'the King of France is bald' is *plainly false*, suggests that although he too was committed to bivalence, he did not derive from this the conclusion that every singular term must denote.[89,90] As we know from OD, Russell found Frege's idea of forcing a denotation on intuitively non-denoting singular terms 'plainly artificial', and he never adopted any version of it. Following the writing of OD, during which Russell saw the potential of treating a notation as an *incomplete symbol*, the introduction of a *notation* was certainly not tantamount, for him, to the introduction of a realm of entities.[91] However, Frege warns that if a function did not assign a value to every object, and in particular if a concept (a function to truth values) did not assign a definite truth value to every object, 'it would be impossible to set forth logical laws about them',[92] seemingly in part because the language would contain sentences with no truth value. In fact, Frege seems never to have contemplated the possibility that not every complete expression of logic would denote. This presupposition, that in logic every term must denote, is enacted through the formulation of the logical rule of universal instantiation, one of Frege's Basic laws. As a consequence, any primitive notation that Frege introduces into the logic is fraught with ontological commitment.[93]

Frege triggers this commitment when he introduces the operator that allows him to express the courses-of-values of functions. The introduction of a *notation* for the courses-of-values of functions *was* tantamount, for Frege, to the introduction of a realm of entities. This is the second ontological commitment enshrined in Frege's syntax. Frege was aware of the ontological implications of his notation.

> The introduction of a notation for courses-of-values seems to me to be one of the most important supplementations that I have made of my Begriffsschrift since my first publication on this subject. By introducing

[89] I read this criticism of Frege's theory of sense and denotation as reflecting Russell's pre-theoretical understanding of the sentence. His own theory of descriptions, of course, *makes* the sentence come out false.

[90] In OD and in later work, for example 'The Nature of Truth', Russell expresses his commitment to bivalence but calls it 'the law of excluded middle' (in OD the law is that a sentence or its negation is true). 'The Nature of Truth' is in *CP4*.

[91] See n. 56 and the reference therein.

[92] 'Function and Concept'.

[93] So for Frege, every definite description *denotes*, but this does not imply for Frege (as it does for Russell) that the description is 'proper'.

 

> it we also extend the domain of arguments of any function.[94] [Underlining added.]

Frege's type theory protects against the application of a function to itself through a syntactic prohibition,[95] and so should have protected his theory from the Contradiction. Unfortunately, the courses-of-values give functions of all types a presence in the realm of objects, where they may commingle, thus undoing the syntactical constraints of the type theory. Frege's two ontological ideas, both enshrined in syntax, work against one another.

As objectifications of functions, courses-of-values do a lot of work for Frege. They generate an object, call it 'A', corresponding to the function that assigns *The False* to every object. Then they generate an object, call it 'B', corresponding to the function that assigns *The False* to every object but A. And so on. In this way, the courses-of-values guarantee an infinite supply of objects.

They also allow Frege to truncate his type theory after only two or three levels because the functions that serve as arguments to higher order functions can be represented by their objectifications.

> It may be briefly observed here that this economy [not needing quantification over functions of higher than second-level] is made possible by the fact that second-level functions can be represented in a certain manner by first-level functions, whereby the functions that appear as arguments of the former are represented by their courses-of-values.[96]

Frege explicitly treats second-level functions (functions whose arguments are functions on objects), presumably because both quantifiers (over objects) and (unobjectified) cardinal numbers appear at this level.[97] But the availability of a representation of functions of higher levels by functions of lower levels allowed him to stop at level three and go no further.[98]

[94] *Grundgesetze,* sect. 9. One may question whether the supplement was 'important'; it certainly was consequential.

[95] To be precise, the syntactic prohibition doesn't prevent the act; it only prevents our talking about it. Syntactic prohibitions cannot prevent what happens behind closed doors. My contention that courses-of-values just *are* the functions is one of those behind-closed-doors claims; it cannot be talked about in Frege's language.

[96] *Grundgesetze* sect. 25.

[97] The primitive function that carries first order functions to their objectifications also appears at this level. Frege also takes as primitive the *third* level function required to institute quantification over first-level functions.

[98] Frege announced the availability of this method of representing higher order functions by lower order ones in 'Function and Concept' (1891) even before *Grundgesetze*. He could have stopped at an even lower level, but he may have thought that one finite level is as good as another. As noted, going to level three allowed him to give more intuitive definitions of a number of notions.

 

The contradiction shows that not all of Frege's functions can be objectified. Because of the way that objectification of functions is embedded in Frege's syntax, this forces us back to a fundamental issue about denoting: how, without artificiality, should we treat a denoting phrase like '$\lambda x\, x \notin x$', which intuitively does not denote, in a language in which '$(\exists x)x = \lambda y\, \Phi y$' is provable? Perhaps it was this issue that Whitehead and Russell hoped the present King of France would solve. Of course, even if the Royal intellect had been up to the task, the solution would not, by itself, have solved the contradiction. But it would have clarified the logical situation.

***1.2.6 Eliminating denoting phrases to attack the Contradiction***

As noted, the language of *Grundgesetze* includes among its primitives a notation for an operator that can be applied to an incomplete expression to form a name of the course-of-values of the function denoted by the incomplete expression. '$\lambda x\mathrm{F}(x)$' denotes the course-of-values of the function denoted by 'F(_)'.[99] The notation alone seems to commit us to what may turn out to be dubious entities.[100] One way of bringing such entities under control is to revise the treatment of expressions like '$\lambda x\mathrm{F}(x)$' so that instead of presupposing the existence of the denotation, it asserts it.[101] It is this that Russell's theory of descriptions finally succeeds in doing.

How would Frege's system fare with the notation for the dubious courses-of-values transformed and eliminated in the manner of Russell? This is an endeavour of the speculative imagination, but let's try it. Following Russell's method, we would introduce a new primitive for a relation between an object $y$ and a function $\phi(\xi)$ (read as '*$y$ objectifies* the function $\phi(\xi)$'). We might write something like, '$\mathrm{O}x(y,\phi(x))$' as the new primitive, and then define '$\lambda x f(x)$'.[102]

[99] Keep in mind that the incomplete expression 'F(_)' denotes something unsaturated and that the complete expression '$\lambda x\mathrm{F}(x)$' denotes something saturated (an object).

[100] The worries about the denotation of the denoting phrase 'some man' being a *very paradoxical object* must have been amplified for Russell when he realized that the denotations of denoting phrases like '$\lambda x\, x \notin x$' yield something even more challenging than paradox, provable inconsistency.

[101] Another way would be to allow non-denoting singular expressions. Another way would be to assign an arbitrary object as denotation for singular expressions that do not denote in a natural way. Frege advocates both of these methods in suitable contexts, but he allows neither for '$\lambda x F(x)$'. His fruitless revisions of Axiom V might be viewed as tacit application of the second method, but that is not the way *he* viewed what he was doing.

[102] The reason for the variable binding operator '$\mathrm{O}z$' is that I am trying to follow Frege's dictum that a functional expression can never appear without its argument place filled, but perhaps I shouldn't try. Even Russell states that 'the great difficulty of [Frege's] symbolism' prevented him

 

$\lambda x f(x) =_{\mathrm{DF}} \text{The } y\ \mathrm{O}x(y, f(x))$.

Axiom V should now require that there *be* objectifications of the two functions:

Axiom V$^{\mathrm{o}}$: $(u)(v)[\mathrm{O}x(u, f(x)) . \mathrm{O}x(v, h(x)) \supset ((x)(f(x)=h(x)) \equiv u=v)]$

Axiom V$^{\mathrm{o}}$ immediately implies that there is *at most* one objectification of a function and that no object objectifies distinct functions.[103] But it doesn't imply that any functions are objectified. We would want to replace Frege's definition of '$\in$' with something like:

$x \in y =_{\mathrm{DF}} (\exists f)(\mathrm{O}z(y, f(z)) . f(x))$

In this form, Axiom V$^{\mathrm{o}}$ might be seen as an appropriate focus for attempts to repair the difficulty.[104, 105]

Interestingly, in his initial response to the Contradiction, Frege starts down the path of questioning which functions have courses-of-value. However, he quickly abandons that route and returns to the assumption that every function *has* a course-of-values. He describes what he does as modifying Axiom V to accommodate a slightly reconceived notion of course-of-values. He does not see himself as accepting the view that some functions don't have a course-of-values, and hence that their course-of-values abstract would be non-denoting (or at least, *improper*).[106]

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from immediately grasping its content. As for me, if I stop reading *Grundgesetze* to eat dinner, when I return, I can no longer understand the notation.

[103] Drop '$h$' to '$f$'. Then '$(x)(f(x) = f(x))$' is a logical truth, and drops away. For the second part, drop both '$u$' and '$v$' to '$w$'. Then '$w=w$' drops away.

[104] Here I have tried to follow Russell's general scheme for putting an arbitrary singular term into standard form as a definite description by adding a new relation. If '$x \in y$' holds in the above sense, it holds under Frege's definition of '$\in$' in *Grundgesetze*, but the converse is not true, because in Frege's logic, '$(\exists y)\ y=\lambda x f(x)$' does not imply that the description abbreviated by '$\lambda x f(x)$' is proper. This is due to the fact that, as noted, having a denotation and being *proper* are distinct in Frege's system (unlike Russell's system). An alternative definition of course-of-values abstracts in terms of '$\in$' as a new primitive was discussed in the section on Axiom V (on p. 958) $\lambda x f(x) =_{\mathrm{DF}}$ The $y\ (x)(x \in y \equiv f(x))$. We cannot both define '$\in$' in terms of course-of-values abstraction and define course-of-values abstraction in terms of '$\in$'. So the earlier way of defining course-of-values abstracts assumes that '$\in$' will be taken as a primitive. Building a theory of classes with '$\in$' as a primitive seems like a promising idea. But it may not be a part of *Pure Logic*.

[105] In contrast to my speculations, Urquhart's 'Introduction' provides a detailed historical account of the actual ways in which Russell thought his theory of descriptions was of help in solving the Contradiction.

[106] Frege's initial response is in the Appendix to Volume II; translated in the *Basic Laws*. He writes, 'We must take into account the possibility that there are some concepts [functions to truth

 

### *1.2.7 How to derive Russell's type theory from Frege's* Grundgesetze

A number of alternatives are available for exploration once the issue is seen clearly to be: Which, if any, of the functions *have* an objectification? One alternative is to attempt to discriminate between those functions that have courses-of-values and those that don't. This seemed a promising route, and Russell followed it for a few years producing what he later called the 'zig zag' theory.[107] However, fussing with Axiom V (which occupied Russell and preoccupied Frege) was not a fruitful way of walking that route. Axiom V is simply too deeply embedded in Frege's metaphysical presuppositions. Frege's first attempt to repair the Contradiction was to pin the problem on concepts that apply to their own courses-of-values. So he tried, fruitlessly, to identify courses-of-values in a way that ignored self-applicability. He revised Axiom V to:

$$\text{Axiom V}'\text{: } (x)[x \neq \lambda x f(x) \,.\, x \neq \lambda x h(x) \supset (f(x) \equiv h(x))] \equiv \lambda x f(x) = \lambda x h(x)$$

But this would not suffice.

From a type theoretical point of view, the courses-of-values undo the benefit of the type theory, so the simplest alternative from that point of view is to undo the undoing by doing away with the courses-of-values entirely and saying that *no* functions have courses-of-values. The original justification of the courses-of-values, I argued, was more from metaphysical ideology than from derivational need (which is not to deny their derivational power). This approach amounts to discarding Axiom V as well as the whole syntactical apparatus of course-of-values abstracts.

It is important to recognize that Russellian type theory, said to be Russell's ultimate resolution of the Contradiction, does exactly this. Frege's metaphysical distinction between (unsaturated) functions and their (saturated) objectifications is simply ignored. Cardinal numbers are taken to be essentially[108] the very second-level functions that Frege identified in his *Grundlagen* in terms of equinumerosity of first-level functions.[109] For Russell, the numbers *are* functions; for Frege, it was

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values] having no extension [course-of-values]—at any rate, none in the ordinary sense of the word.' In this, he momentarily entertains the idea that some functions may lack a course of values, and then quickly reverts to the idea that what he needs to do is modify the identity conditions for course-of-values.

[107] The logical/historical situation is beautifully expounded in Alasdair Urquhart's 'Introduction' and in his 'Logic and Denotation' (forthcoming).

[108] The main difference being that where Frege's functions are to truth values, Russell's functions are to propositions. See also n. 110.

[109] In *Grundgesetze* these second-level functions never quite appear because Frege keeps representing the first-level *concepts* (as he calls them) by their objectified courses-of-values. This is done

 

unthinkable that numbers should not be objects. Let us look at some consequences of the abandonment of courses-of-values.

There would be no way to guarantee an infinite supply of objects, so a separate Axiom of Infinity for objects would be required. In addition, we could not truncate the hierarchy of higher order functions because we could no longer count on being able to represent higher level functions by lower level ones through the objectifications of the arguments of the higher level functions. So an endless hierarchy of types, with new entities at each level, would ensue. We would now be required to identify the cardinal numbers with the (unobjectified) second order functions, but unhappily, a distinct claimant to be that cardinal number would appear at each higher level.

These are among the familiar troubles of type theory. They plagued Russell when he abandoned Frege's course-of-values of a function and struck out with just the functions themselves. Once courses-of-values are eliminated, *Grundgesetze* transforms into type theory.[110]

## Part 2: 'On Denoting'

### *2.1 Syntax and the Semantical Agenda*

#### *2.1.1 When is a phrase denoting?*

Turning finally to OD itself, the paper opens with examples of what Russell means by 'denoting phrase'. The examples lean heavily, but not exclusively, on definite descriptions. He then says, stylishly, but confusingly,

Thus a phrase is denoting solely in virtue of its [syntactical] *form*.[111]

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consistently throughout the development of the theory of numbers in *Grundgesetze* (see his statement of intention at the beginning of section 34) and done in a way that adds considerably to the opaqueness of the notation. One finds oneself appreciating Russell's generally transparent, though unsystematic, notational ambiguity.

[110] Of course, this is not quite *Russell's* type theory. Russell considered Frege's incorporation of *The True* and *The False* as distinguished objects to be unnatural. The predicates that Frege took to denote functions from objects to these truth values, Russell took to stand for functions from objects to propositions. Russell's type theory is an *intensional* theory in that his basic entity is a *propositional* function rather than a truth valued function. So for Russell, it made sense to speak of the *extension* of a propositional function, where these extensions had a different principle of individuation than the propositional functions. This distinction is quite different from Frege's distinction between saturated and unsaturated entities (which do not have distinct principles of individuation). I suggested earlier that there are natural surrogates for Frege's truth valued functions that might be taken as the extensions of Russell's propositional functions. I have also been talking as if Russell's type theory was more like that of Ramsey (1925). The surrogates for Frege's truth valued functions definitely exist in Ramsey's theory; I'm less certain that they exist in Russell's.

[111] OD para 1.

 

This is an announcement of a fundamental change from the doctrine of Chapter V of *PoM* and indeed from almost all of Russell's unpublished, intervening writing right up to the few days in July 1905 in which he wrote OD. In *PoM*, a phrase was said to be a 'denoting phrase' because it was a phrase that denoted. Now, a phrase will be said to be a 'denoting phrase' whenever it has the appropriate syntactical form. No semantic matters are presupposed. Russell's use of 'denoting phrase' will now be like that of the linguist's 'noun phrase' or 'verb phrase', but whereas the linguist's terminology is explicitly drawn from the language of grammar, the word 'denoting' is drawn from semantics.

The sentence quoted lends itself to the misinterpretation that the semantic issue of whether or not a phrase denotes is determined solely by its syntactical form. This is *not* what Russell is saying. What he is saying is that he will call a phrase a 'denoting-phrase' (I briefly add a hyphen to emphasize that this is an indecomposable idiom) solely on the basis of grammatical considerations. To drive home the point that whether 'a phrase is denoting' is independent of semantical considerations, he then mixes this use with the use of 'denote' and its cognates in their original semantical sense, and remarks,

> We may distinguish three cases: (1) A phrase may be denoting, and yet not denote anything; *e.g.,* "the present King of France". (2) A phrase [may be denoting, and] may denote one definite object; *e.g.,* "the present King of England" denotes a certain man. (3) A phrase [may be denoting, and] may denote ambiguously; *e.g.,* "a man" denotes not many men, but an ambiguous man.[112]

This sounds like a declaration, but it isn't. What he means is that the following three claims *seem initially plausible*: that some denoting-phrases fail to pick out anything, that some denoting-phrases pick out a unique thing, and that some denoting-phrases either pick things out ambiguously or pick out an ambiguous thing.[113] He explored the third case as far back as *PoM*, where he explicitly rejected ambiguous denoting and perhaps the ambiguous man as well.[114] So Russell's three cases are not the announcement of settled views that they appear to be, but rather an intuitive introduction to the subject.

[112] OD para 1.

[113] Russell suggests, carelessly, that my 'or' should be 'and' by writing 'e.g.'.

[114] The question in *PoM* is whether Russell took his 'very paradoxical object'—the disjunction of all men—to be the ambiguous man. In any case, since such expressions as 'a man' cease to denote in OD, Russell certainly is not, at this point, *declaring* that 'a man' denotes an ambiguous man.

 

It does seem like a shaky start to one of the most widely read pieces of philosophical writing by one of the greatest philosophers of the twentieth century, but then, when Russell, at the age of 33, wrote it in twelve days at the end of July 1905 (with time off for a three day vacation and for mourning the death of a close friend), he may not have known that he would be one of the greatest philosophers of the twentieth century.[115]

### *2.1.2 The* Principle *of the new theory of denoting*

In order to assess the widely quoted second paragraph of OD, we must glance ahead to the *principle* of the theory of denoting that Russell is going to advocate. It is this: that denoting phrases have no meaning of their own (he usually says 'no meaning in isolation'), although they do, of course, affect the meaning of the whole sentence in which they occur. Given that, for Russell, the meaning of a sentence is a proposition, what this amounts to is the claim that a denoting phrase does not contribute a single element (its own meaning) to the proposition in the way that, say, a proper name contributes the individual named to the proposition. A denoting phrase might contribute the various meanings of its constituent words as well as some structural features to the proposition, but unlike the proper name, there is no single element, no single complex of meanings, that plays the same structural role in the proposition as the grammatical role that the denoting phrase plays in the sentence.

This is a surprising view, because it is roughly true that in English, wherever a name can occur in a grammatically correct sentence, a denoting phrase can also occur.[116] This suggests that the propositions that are expressed by English sentences should have an analogous structural feature: wherever the meaning of a name (for Russell, the thing named) can occur in a proposition, the meaning of a denoting phrase should also be able to occur. In fact, we might expect that when we replace a name by a denoting phrase in an English sentence, the resulting proposition is obtained by an analogous operation: we replace the thing named by the meaning of the denoting phrase. Russell is rejecting this hypothesis in a very radical way. He is not only saying that the apparent similarity of grammatical structure between sentences containing names and those containing denoting phrases is not matched by a similarity in structure at the propositional level; he is saying that denoting phrases don't *have* any meaning that *could* play such a role.[117]

[115] Or maybe he did know.

[116] 'Harry met Sally', 'A man met a woman', 'Some man met every woman', 'Every man met the woman', etc.

 

Russell puts this rather clearly with respect to what he calls 'the most primitive of denoting phrases'.

> *Everything*, *nothing*, and *something*, are not assumed to have any meaning in isolation, but a meaning is assigned to *every* proposition [he means *sentence*] in which they occur.[118]

And then he repeats it more awkwardly.

> This is the principle of the theory of denoting I wish to advocate: that denoting phrases never have any meaning in themselves, but that every proposition in whose verbal expression they occur has a meaning.[119]

These *meanings in isolation* that are said not to exist are exactly the *denoting concepts* of his own *PoM*.[120] Denoting concepts are now banished.

Russell is very aware of the fact that he is opposing his own former views regarding denoting concepts. At the first mention of the theory of denoting he intends to advocate, Russell inserts a footnote.

> I have discussed this subject in *Principles of Mathematics*, chapter v., and § 476. The theory there advocated is very nearly the same as Frege's, and is quite different from the theory to be advocated in what follows.[121]

[117] Russell's claim that grammatical structure is not a correct guide to *logical* structure (the structure of what is meant) and his apparent solutions to puzzles using the hidden logical forms he unearthed was surely a prime mover of the project of trying to solve philosophical problems through the 'correct' logical analysis of systematically misleading grammatical forms. As logic and linguistics grow closer together it may turn out that what misled us was not an incorrect understanding of logical form but an inadequate understanding of *grammatical* form.

[118] OD para. 4. The talk of *assigning* a meaning to the sentences in which denoting phrases occur is meant to indicate that his theory will show *how* the meaning of the sentence depends, in a systematic way, on the meaningful elements *within* the denoting phrase.

[119] OD para. 4. Here, the word 'proposition' must be given first, the meaning *meaning of a sentence*, and then the meaning *sentence* to prevent the second clause from being nonsense, and even then it becomes a tautology. This is one of those places at which one wishes that Russell had spent a few more days editing his hastily written manuscript.

[120] As noted, the *denoting concepts* of *PoM* are also central to a series of unpublished papers on *meaning* and *denotation* that Russell wrote between *PoM* and OD.

[121] OD n. 1. The theory of *PoM* is like Frege's primarily in that definite descriptions have both a meaning (in isolation) and a denotation, and that the meaning (for Russell, a denoting concept; for Frege, a sense) is also said to have a denotation. For Frege, the meaning is the primary vehicle of denotation, and the same holds for Russell. 'Thus it is the meaning, not the name, which denotes the denotation; and denoting is a fact which concerns logic, not the theory of language or of naming.' ('On Meaning and Denotation' p. 318 in *CP4*). However, the theories differ in that for Russell the *rule* is that propositions are about their own constituents. *Denoting* brings in a new and peripheral form of representation, one he rids himself of in OD. For Frege the rule is that thoughts (Frege's analogue to propositions) are about the Bedeutung (Frege's analogue to denotation) of their constituents. Bedeutung is central to the explanation of how a thought can be about something in the world.

 

Furthermore, although the bulk of OD is devoted to showing how well we can get along without denoting concepts, the long and difficult Gray's Elegy passage is a head-on attack on denoting concepts. It purports to show that there could not be such things, or, if there were, they would have unacceptable properties (such as being unnameable).[122, 123]

The *principle* of Russell's new theory is that denoting phrases have no meaning in isolation. But the *project* of OD is to show how to get along without these meanings, that is, how *does* the meaning of a sentence containing a denoting phrase depend on the denoting phrase (as it obviously does).

The project will be carried out in two steps. Russell will first give his analysis of sentences containing denoting phrases involving the determiners 'every', 'some', and 'no', for example 'everything', 'something', 'nothing', 'every man', 'some man', 'no man'. He does this without appealing to denoting concepts. The first two correspond loosely to the quantifiers of modern logic (sometimes read as 'every $x$ is such that' and 'there is an $x$ such that'); the third is usually taken as a compound in modern logic ('it is not the case that there is an $x$ such that').[124] It is interesting to note that of the six determiners closely studied in chapter five of *PoM*, each of which was said to differ semantically from the others, four are omitted and one, 'no', is added. This sort of 'starting afresh' was not unusual for Russell, who often indicated his awareness of difficulties in views he was advocating. It was probably also a product of the extraordinary ease and fluidity with which he wrote.[125]

[122] The Gray's Elegy argument is often said to be an argument against Frege, but I think not. Russell argues separately against the specifics of Frege's theory. Russell's unpublished writings between 1903 and 1905 show him ruminating on his own theory of denoting concepts, which are an anomaly within Russell's semantics, and those writings also show the origin of the Gray's Elegy argument, which appears in the course of those ruminations. The argument is founded on the principle that aside from denoting concepts, propositions are about their own constituents. Frege's propositions are *never* about their own constituents (except by a fluke). This argument is Russell versus Russell, not Russell versus Frege. I have indicated my own analysis of, and my scepticism about, Russell's argument in Appendix C to my 'Opacity' (1986). I will explain my view more thoroughly in a sequel to this article.

[123] If Russell's argument were correct, denoting concepts would be the first *stubborn objects* in the sense of Quine (1960).

[124] These 'readings' make no claim to exactness, nor even rough correctness, but they may be useful to those who have had contact with modern logic.

[125] *CP4* contains 640 printed pages of Russell's writing during a three year period. It omits all of his social and political papers during the period as well as his very extensive and detailed correspondence. The original manuscripts are written by hand. In 1950 Russell won the Nobel Prize for Literature 'in recognition of his varied and significant writings in which he champions humanitarian ideals and freedom of thought'. No doubt the Nobel Committee was impressed with Russell's social and political thought, but his writing style also must have contributed to the literature prize.

 

In the second step, which occupies more than 80% of the article, Russell turns to definite descriptions.[126] In this case, the project will be guided by a method of transforming sentences containing definite descriptions into putatively equivalent ones in which the definite descriptions no longer explicitly appear. The method makes use of the analyses given in the first step, and thus reduces the problem of analyzing definite descriptions to the already solved problem of analysing the quantificational denoting phrases. Russell writes, 'The above gives a reduction of all propositions [sentences] in which denoting phrases occur to forms in which no such phrases occur'. This reduction of sentences containing definite descriptions to quantified forms in which no corresponding element appears is now commonly referred to as the *elimination* of definite descriptions.

For one familiar with the language of modern first-order logic, the project would have been more perspicuously presented had Russell used that language as an intermediary,[127] first translating sentences from the denoting phrase language (English) into the language of modern logic (as logic students are presently taught to do), and then describing the semantics of this intermediary language in terms of propositional functions and their properties. But in OD, Russell cuts out the middle man and describes how to assign propositions directly to sentences of the denoting phrase language. This adds considerable difficulty to the text.[128]

## *2.2 Knowing and Denoting*

### ***2.2.1 The epistemological detour***

The second paragraph appears as an epistemological detour on the way to the analysis of denoting phrases. It has the tone of an advertisement for the importance of denoting, in the manner of the *PoM* advertisement quoted above ('This notion lies at the bottom of all theories of substance, of the subject-predicate logic,' etc.).

> The subject of denoting is of very great importance, not only in logic and mathematics, but also in theory of knowledge. For example, we know that the centre of mass of the Solar System at a definite instant is some definite

[126] As noted, these are barely touched on in *PoM*.

[127] Some, incorrectly, read him as doing that indirectly, but alas he does not. He does provide a few English formulations that try to capture the scope differences that are so perspicuously displayed in logical notation.

[128] Russell talks as much of propositional functions as he does of propositions, but that talk is best seen, I believe, as part of a metalinguistic description of the proposition expressed. I will come back to this when I discuss his theory.

 

> point, and we can affirm a number of propositions about it; but we have no immediate *acquaintance* with this point, which is only known to us by description. The distinction between *acquaintance* and *knowledge about* is the distinction between the things we have presentations of, and the things we only reach by means of denoting phrases. It often happens that we know that a certain phrase denotes unambiguously, although we have no acquaintance with what it denotes; this occurs in the above case of the centre of mass. In perception we have acquaintance with the objects of perception, and in thought we have acquaintance with objects of a more abstract logical character; but we do not necessarily have acquaintance with the objects denoted by phrases composed of words with whose meanings we are acquainted. To take a very important instance: There seems no reason to believe that we are ever acquainted with other people's minds, seeing that these are not directly perceived; hence what we know about them is obtained through denoting. All thinking has to start from acquaintance; but it succeeds [through denoting] in thinking *about* many things with which we have no acquaintance.[129]

It is clear that Russell considered these matters important because they begin and end the article. He chose to end his article by asserting that his theory had vindicated the preceding claims.

> One interesting result of the above theory of denoting is this: when there is anything with which we do not have immediate acquaintance, but only definition by denoting phrases [he means, by singular denoting phrases], then the propositions in which this thing is introduced by means of a denoting phrase do not really contain this thing as a constituent, but contain instead the constituents expressed by the several words of the denoting phrase. Thus in every proposition that we can apprehend (*i.e.* not only in those whose truth or falsehood we can judge of, but in all that we can think about), all the constituents are really entities with which we have immediate acquaintance. Now such things as matter (in the sense in which matter occurs in physics) and the minds of other people are known to us only by denoting phrases, *i.e.*, we are not acquainted with them, but we know them as what has such and such properties. Hence, although we can form propositional functions $C(x)$ which must hold of such and such a material particle, or of So-and-so's mind, yet we are not acquainted with the propositions which affirm these things that we know must be true, because we cannot apprehend the actual entities concerned. What we know is 'So-and-so has a mind which has such and such properties' but we do not know 'A has such and such properties,' where A is the mind in question. In such a case, we know the properties of a thing without having acquaintance with the thing itself, and without, conse-

[129] OD, para. 2. To my knowledge, this is the first *published* statement of an epistemological principle that will become a staple of Russell's thought, 'All thinking has to start from acquaintance; but it succeeds [through denoting] in thinking *about* many things with which we have no acquaintance.'

 

> quently, knowing any single proposition of which the thing itself is a constituent.[130]

There are several claims here. First, that the propositions expressed by sentences containing singular denoting phrases (henceforth, I will just say *definite descriptions*)[131] need only contain the meanings of the meaningful parts of the denoting phrase and need not contain the denotation itself, second, that this allows us to limit the constituents of the propositions we entertain (Russell says *apprehend*) to 'entities with which we have immediate acquaintance', and third, that through the use of definite descriptions we can have *knowledge about* things with which we are not acquainted.

This addresses the worry that if the denotation itself had to be an element of any proposition expressed by a sentence containing a definite description, then entities we could *describe* but could not plausibly be acquainted with would creep into the proposition, challenging our ability to entertain it. And this, in turn, would conflict with our seeming ability to understand such sentences (i.e. know their meaning). Worse, if the denotation *replaced* the meaning, informative identities, like 'Scott is the author of *Waverley*', would be transformed into trivialities.

Russell is not here suggesting that there is anything ontologically insecure about other minds or space-time points.[132] He does not think that they exist merely through description (as he seemed to think of mythological characters). Their existence is just as secure as that of the middle-sized material objects that we perceive. They are just too inaccessible or too small to present themselves directly.

In fact, although Russell's theory of descriptions is often described as a model for avoiding ontological commitments, it is essentially neutral with respect to ontological commitment.[133] This, I think, is one of its

[130] Penultimate paragraph of OD.

[131] Singular denoting phrases are those that purport to pick out a single individual. Russell often uses genitive constructions, like 'Smith's wife', interchangeably with true definite descriptions like 'the wife of Smith'. Russell believed that all singular denoting phrases were easily rewritten as definite descriptions, and that such rewriting had no semantic, logical, or epistemological effect. He writes, '[a singular denoting phrase] is always, in its logical essence, though not necessarily in linguistic form, compounded of a phrase which only has meaning [for example a noun or noun clause], together with the word *the*.' ('On the Meaning and Denotation of Phrases' p. 285.) So, for Russell, the task of analysing singular denoting phrases reduced to the task of analysing definite descriptions. I will play along with Russell, and help to fix ideas, by pretending that all singular denoting phrases *are* definite descriptions.

[132] In this respect, Russell's treatment of definite descriptions contrasts sharply with his later treatment of *virtual* extensions of propositional functions (see p. 952).

[133] As noted, this is not true of the generalized *principle of denoting* that Russell educed from his work on descriptions (see p. 951).

 

virtues. Meinong believed that there is a non-existent object that is both round and square. Russell didn't. This is an ontological dispute. If Meinong is right, and nothing else is round and square, then the definite description 'the round square' denotes, and there is no way of using Russell's theory of descriptions to remove this object from the ontology. If Meinong is wrong, then the definite description doesn't denote, and that's the end of it. The one place where Russell's theory of descriptions disagrees with Meinong's, once the prior ontological issues are settled, is that Meinong believed that there is a second non-existent object that is triangular and round and square. And so, for Russell, the description 'the round square' would fail to denote because of an excess of round squares. Note that this denotation failure does not in any way reduce the ontology. But it does show that they had different theories of the semantics of definite description.

To those of us raised on Frege or on Russell's theory of descriptions (introduced in OD), it will seem odd to think that the denotation could be a part of the meaning of a definite description. But this was a real worry for Russell. He can be seen addressing it again and again in various papers leading up to OD.[134] Furthermore, as Frege explains in S&B, it was exactly the worry that informative identities would become trivialities that drove him, in section 8 of his *Begriffsschrift* to the extreme view that identity, unlike all other relations that are not explicitly about language, was a relation between the linguistic phrases themselves. The view that the denotation is a constituent in what is asserted when a definite description is used has persisted in the work of P. F. Strawson, who, in 1950, defended the view that when we assert 'France's greatest soldier died in exile' the individual referred to is a constituent of the assertion.[135]

[134] For example, in 'Points About Denoting', point (2) is 'When a denoting phrase occurs in a proposition [sentence], does that which is denoted form a constituent of the proposition or not?', and in 'On Meaning and Denotation' he characterizes his view as having the 'strange consequence' that 'we may know a proposition about a man, without knowing that it is about him, and without even having heard of him'.

[135] According to Strawson, there is a *presupposition* whose meaning we may take to be a complex of the meanings of the several words of the description, but *what is asserted* contains the denotation. Strawson would not say 'denotation' but 'individual referred to'. This, at least, *seems* to be what he intended if he is not merely reinventing Frege's theory. I should note that I have considerable sympathy for something like Strawson's idea, especially as modified and elaborated by the work of Keith Donnellan, that the denotation of a definite description can be a part of what is thought or said. Donnellan (1966) calls attention to those cases in which we formulate a definite description intending to describe an individual we already have in mind. I believe this to be the most common use of definite descriptions. However, in the present paper, I am trying to present my discussion and analysis in a way that is recognizably Russellian. Donnellan's 1966 paper is the first in a series.

 

*Knowledge about* and *knowledge by description* (as Russell came to speak of it) involves only one kind of denoting phrase, the definite description.[136] We have knowledge by description when (and *only* when) we know that there is exactly one individual satisfying the description. The more important notion is *knowledge about*. We have knowledge *about* an individual when our knowledge is expressed using a sentence containing a definite description that denotes that individual. As Russell says in OD, we have *knowledge about* 'the things we only reach by denoting phrases'.

### *2.2.2 An oddity*

In the case of definite descriptions, the project of showing how to get along without assigning a separate meaning to denoting phrases is guided by Russell's method for eliminating definite descriptions. So a primary aim of OD is to establish that whenever we have knowledge about an individual, the very same piece of knowledge can be expressed without using a definite description. Thus, someone who speaks a language not containing definite descriptions may express the same propositions as those expressed by one who uses definite descriptions. Suppose one speaker would express his knowledge using a definite description and another speaker would express exactly the same piece of knowledge (the same proposition) without using a definite description. Is the piece of knowledge itself, independently of how the person might put it into words, that is, independently of whether the person reached it through a denoting phrase, a piece of *knowledge about* an individual or not?[137]

There is something very odd about urging the epistemological importance of *denoting*[138] at the beginning of a work whose purpose is to show that the propositions we entertain when we *know, judge, suppose,* etc. contain *no* denoting elements.

Denoting is almost entirely absent from the OD theory. Its only presence is through the almost off-hand remark that if a proposition of the

[136] Remember the pretence of n. 131.

[137] This difficulty is amplified by the fact that Russell didn't stick with a single logical form for sentences containing definite descriptions but gave many different (though equivalent) forms. A thinker may be unaware that the proposition entertained is equivalent to a proposition expressible using a definite description. In *PM*, he finally gives a standard form for the elimination of definite descriptions when accompanied by scope indicators. (See Ch. 3 'Incomplete Symbols' of the Introduction to *PM*.)

[138] 'The subject of denoting is of very great importance, not only in logic and mathematics, but also in theory of knowledge.'

 

form '*x* is identical with the F' is true (where the definite description[139] is eliminated in accordance with the theory):

> We may then say that the entity *x* is the denotation of the [definite description].[140]

And even here, Russell is quick to deflate.

> [that which has denotation] will be merely the *phrase*, not anything that can be called the *meaning*.

Denoting has been reduced to a property of proper definite descriptions, linguistic accidents that have no counterparts in the realm of thought. It sidles into the OD picture through mere (and meaningless!) linguistic phrases. An ignoble end for a notion 'of very great importance'.[141,142]

To the best of my knowledge, Russell never felt any tension between urging the epistemological importance of *denoting* and his banishment of denoting elements from propositions. He continued to the end of his life to happily emphasize the importance of knowledge by description while insisting that descriptions contributed no unique element to the propositions that are the objects of knowledge.

### *2.2.3 Whence, knowledge by description and acquaintance?*

The distinction between knowledge by acquaintance and knowledge by description feels like a hangover from the *PoM* theory of denoting concepts, where it would have made more sense. In *PoM*, a definite description *had* a meaning in isolation, and the meaning (a denoting concept) had a denotation. There, the piece of knowledge itself was marked by the presence (or absence) of an identifiable denoting concept, so the distinction between knowledge expressed using a definite description and knowledge expressed without such was robustly represented in the objects of thought (propositions). As it turns out, that *is* where the distinction was born. Not in *PoM*, but in the hitherto unpublished 'Points

[139] Russell says 'denoting phrase' but his remark, 'there cannot be more than one such [*x*]' makes it clear that only singular denoting phrases, which we are pretending to be definite descriptions, are at issue.

[140] OD, p. 488.

[141] Presumably, the introduction of this very thin notion of *denoting*, isolated from the body of the logical and semantic theory, is simply to avoid cutting *knowledge about* completely out of the picture. But see n. 162 below.

[142] I have suggested that Russell need not have given up a 'meaning in isolation' for definite description to achieve the results his elimination of descriptions achieved, by which I mean that we can separate Russell's *theorems* of descriptions—the equivalences given by his method of elimination—from his *theory* of descriptions—the semantic theory that banishes denoting concepts. The claim is argued in 'What is Russell's Theory of Descriptions?' p. 244.

 

About Denoting', written more than a year before OD.[143] Here, *before* Russell abandoned denoting concepts, are the roots of the distinction between knowledge by acquaintance and knowledge by description, as well as the roots of the *epistemological principle* 'All thinking has to start from acquaintance; but it succeeds [through denoting] in thinking about many things with which we have no acquaintance.'[144] I call the first half of this, that all thinking has to start from acquaintance, the *principle of acquaintance.*

> … if I ask: Is Smith married? and the answer is affirmative, I then know that "Smith's wife" is a denoting phrase [i.e. a phrase that *does* denote], although I don't know who Smith's wife is. We may distinguish the terms [i.e. individuals] with which we are *acquainted*[145] from others which are merely denoted. E.g. in the above case, I am supposed to be acquainted with the term [individual] Smith and the relation *marriage,* and thence to be able to conceive a term [individual] having this relation to Smith, although I am not acquainted with any such term [individual]. … we know that every human being now living has one and only one father. … Nevertheless, it's a wise child etc. This shows that to be known by description is not the same thing as to be known by acquaintance, for "the father of $x$" is an adequate description in the sense that, as a matter of fact, there is only one person to whom it is applicable. … It is necessary, for the understanding of a proposition, to have *acquaintance* with the *meaning* of every constituent of the meaning, and of the whole [better, *acquaintance* with every constituent of the proposition, each of which may be taken to be a meaning]; it is not necessary to have acquaintance with such constituents of the denotation as are not constituents of the meaning.[146]

Russell's distinction presupposes the idea that the elements of any proposition we can entertain must be, in some sense, *known to us*. I believe that he called that sense of knowing, *acquaintance.*[147] If the presupposition is correct, this makes the principle of acquaintance tautological, as I think it should be for Russell. *Acquaintance* is a sense of *know* that takes an object as direct object, know *it,* not know *that.*[148]

[143] In *CP4*.

[144] The editor's excellent headnote to the article notes that these topics are covered.

[145] This may be Russell's first use of *acquaintance* in his special epistemological sense. He almost invariably used it to contrast with knowledge *about*, which is knowledge by description.

[146] From points numbered (1) and (5) in 'Points About Denoting' (1903) in *CP4*. The last is not the most graceful formulation of the epistemological principle, but it is unmistakable.

[147] In this way of looking at the notion, the sorts of things we are acquainted with then becomes a matter of one's epistemological *theory.*

[148] In 'Knowledge by Acquaintance and Knowledge by Description', Russell distinguishes *knowledge of things* from *knowledge of truths*. The article, hereafter KA&KD, first appears in *Proceedings*

 

In this pre-sense-data era, Russell obviously took it that one could be acquainted with an external individual like Smith, perhaps as an 'object of perception'. Thus, in a natural way, one would be acquainted with one's acquaintances. So I will take perception as the paradigm for acquaintance with external material objects. If we do so, the notion that one might be acquainted with Smith, but not his wife, has all the common sense plausibility that Russell seems to have assigned it.[149] It was only after falling under the baleful influence of sense-data theory that Russell came to believe that the objects of perception were *internal*.

***2.2.4 Is knowledge by description exclusive of knowledge by acquaintance?*** Russell often writes as if there were an opposition between knowing a particular individual *by description* and knowing that individual *by acquaintance*.[150] In particular, he writes as if *not knowing who* a particular description denotes amounts to *not being acquainted* with the individual. But he should not, and almost surely does not, intend that. The point is only that knowledge by description does not *require* knowledge by acquaintance.

In the example, Russell did not *know who* Smith's wife was in the sense of knowing no informative truth of the form, 'Smith's wife = _'.[151] He certainly did not know that Smith's wife was Triphena (as she turns out to be), but this does not show that he was not acquainted with her. One can imagine Russell being teased, 'Do you know who Smith's wife is?' 'No, though I knew that he was recently married.' '*Your* doctor!' 'Amazing! My doctor? Triphena? Are you kidding me?'

Russell may have been able to 'affirm a number of propositions about' Smith's wife,[152] for example, that Smith's wife lives in a grand house and that she has a kind and indulgent husband, but in all such

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*of the Aristotelian Society* 11 (1910), pp. 108–28, and then, in slightly different form as chapter five of Russell's *The Problems of Philosophy* (Russell 1912); hereafter *Problems*. The first version seems more closely connected with OD, whereas the second has more hard-core epistemology.

[149] It is explicitly stated in the material quoted from 'Points About Denoting' that one could be acquainted with Smith. It is thus reasonable to conclude, contrary to some readers of OD, that this view prevailed through 1905, and that the 'objects of perception' in the OD passage were the normal, external, material objects we perceive.

[150] The notions frequently appear together, almost always contrastively.

[151] *Knowing who* intervenes in a way that confuses matters. It is neither *acquaintance* nor *knowledge by description*. In 'Opacity' (1986) I emphasized its contextual basis. We can point at someone and ask, 'Do you know who that is?'

[152] Here, I use the language of the second paragraph of OD.

 

affirmations, Triphena would be known only 'by description'.[153] He might also have been able to affirm a number of propositions about her through acquaintance, for example, that she is an excellent physician and that she takes her patients promptly. It seems obvious that knowledge by description is quite independent of knowledge by acquaintance. In fact, in Russell's most famous example, George IV wishing to know whether Scott was the author of *Waverley*, knowledge by acquaintance and knowledge by description of the same individual are combined in a single thought.

In some cases (other minds, elementary particles, space-time points) the objects themselves may, by their nature and ours, be inaccessible to us. Then we cannot be acquainted with them and have only knowledge by description to fall back on. But this does not affect the general point about the compatibility of the two notions.

### *2.2.5 Rethinking* knowledge by description

I think there is a way of reconstructing the notion *knowledge by description* that helps to resolve the oddity. Russell emphasizes two points: First, that knowledge by description is based on knowledge of truths, and second that the descriptions involved are *definite* descriptions.

> Knowledge of things by description … always involves, as we shall find in the course of the present chapter, some knowledge of truths as its source and ground … This is a matter which is concerned exclusively with *definite* descriptions.[154]

Now suppose that instead of insisting that definite descriptions were essential to knowledge by description he had emphasized *indefinite* descriptions, or better, in the language of logic, *existential generalizations*. The truths that knowledge by description is based on always involve existential generalizations anyway, typically, something like 'there is something such that it, and it alone, is a so-and-so'. Suppose that instead of writing,

> What I wish to discuss is the nature of our knowledge concerning objects in cases where we know that there is an object answering to a definite description, though we are not acquainted with any such object. This is a matter which is concerned exclusively with *definite* descriptions … We shall say that we have "*merely* descriptive knowledge" of the so-and-so when, although we know the so-and-so exists, and although we may possibly be acquainted with the object which is, in fact, the so-and-so, yet we do not know any proposi-

[153] Remember that although 'Smith's wife' is a genitive construction, we are pretending that it is a definite description.

[154] KA&KD, 1912 version.

 

> tion "*a* is the so-and-so", where *a* is something with which we are acquainted.[155]

Russell had written,

> What I wish to discuss is the nature of our knowledge concerning objects in cases where we know that there are one or more objects answering to a description, though we are not acquainted with the objects … We shall say that we have "*merely* descriptive knowledge" of the so-and-so's when, although we know the so-and-so's exists, and although we may possibly be acquainted with some or all of the objects which are, in fact, the so-and-so's, yet we do not know any proposition "$a_1$, $a_2$, … $a_n$ are the so-and-so's", where $a_1$, $a_2$, … $a_n$ are things with which we are acquainted.

Note that knowing the truth of an existential generalization involves knowing that *there are* things of a certain kind, though we may not be acquainted with those things. In the language of knowing-who, which Russell sometimes carelessly uses as if it were interchangeable with acquaintance, knowing the truth of an existential generalization involves knowing that *there are* things of a certain kind, though we may not know exactly which things they are. Thus I may know that there are 29 counties in Utah,[156] without knowing what they are and without ever having travelled to Utah. I may know that there are two authors of *PM* without knowing who they are and certainly without being acquainted with them. I may know that there are assassins among us without knowing exactly who they are, though I may be acquainted with them. Such knowledge by description can even be put into the form of a definite description, provided we allow plurals. I don't know what *the 29 counties in Utah* are. I don't know who *the two authors of PM* are. I don't know who *the assassins* are. I may even *wish to know* whether Fred and Ethyl are the assassins.

My notion depends on logical features of the proposition known, not on features of a 'mere phrase'. Thus it is robust in the way in which Russell's notion was robust in the pre-OD days, when there were still denoting concepts in propositions.

Because, as I have claimed, Russell's theory of descriptions simply takes definite descriptions to be a special case of indefinite description, namely, indefinite descriptions with uniqueness added, and because Russell takes sentences involving indefinite descriptions to have a logical form involving existential generalizations, we can see Russell's knowledge by description as a special case of mine. I would even agree

[155] KA&KD, 1910 version.

[156] Because Alonzo Church has asserted it. I'm counting numerical existential quantifiers such as this as a generalized form of existential generalization.

 

that through knowledge by description we succeed in thinking *about* many things with which we have no acquaintance. But I hesitate to say that it is through *denoting*.[157]

One might restore knowledge through denoting by taking my plural definite descriptions seriously and giving the phrase a denotation.[158] Researchers in the tradition of George Boolos (1984) are pursuing the line of thought that such a denotation would itself be plural. Alternatively, one might restore a more robust form of denoting by following a suggestion of Part 1 of the present paper and treating the propositional functions that inhabit Russell's propositions as denoting their extensions.[159] I will not pursue either line here, but I think that both can be made to work, and both would, I believe, capture an intuitive notion of *knowledge by description* and *knowledge about*.

### *2.2.6* Thinking about *requires knowledge*

There may be another way to restore robustness to *knowledge about*.

Russell's very notion of knowledge by description seems to require an unmentioned complication in his analysis. Knowledge by description requires knowledge that there is exactly one thing satisfying the description. But a description may be used in expressing attitudes other than knowledge, for example, when George IV asked, 'Is the author of *Waverley* present in this room?' Now this is a *query about* Scott in just the way that George IV *knowing* that the author of *Waverley* was present in the room would be *knowledge about* Scott. Russell says that through denoting we succeed in *thinking* about many things, not just *knowing* about many things. So I read Russell as claiming that *knowledge by description* is what enables all forms of *thinking about*. Keep in mind that knowledge of a thing by acquaintance and knowledge of a thing by

[157] 'Knowledge through denoting' is a form of indirect knowledge *of* an individual by way of knowledge *that* there is exactly one thing satisfying a description. The knowledge *that* there is exactly one thing satisfying a description enables us to use a definite description to talk and think (but for the 'oddity') about the individual *of* whom we have indirect knowledge. So the existence of a denoting phrase (or better, a denoting concept, as in the pre-OD days when knowledge through denoting first came on the scene) is critical for knowledge 'through denoting'. My hesitation is born from the lack of a phrase that we would not hesitate to describe as a denoting phrase.

[158] I assume that if there are no assassins among us 'the assassins among us' would have no denotation, as opposed to having a null or empty denotation. Thus, in the case of an empty plural noun clause, like 'flying pigs', I am *not* treating the plural definite description 'the flying pigs' as denoting the extension of the noun clause, here, the null set, instead I am treating it as lacking a denotation. In all other cases, it seems all right to treat the definite description as denoting the extension of the noun clause. The Pluralists want the plural definite description to denote the contents of the extension. That would work for me, because the null set has no content.

[159] See p. 957. This suggestion is again subject to the concerns of n. 158. We don't want the proposition *that there are flying pigs* to be about the null set; we want it to be about nothing.

 

description are Russell's two ways for thought to connect with things. There are, for Russell, no relevant further ways for thought to connect with things. For example, merely *suspecting* that there is exactly one so-and-so does not enable you to think *about* that thing (even assuming your suspicion to be correct). Thought connects directly with things through acquaintance and indirectly through knowledge by description (knowing that there is exactly thing of a certain kind). Mere suspicion won't hook you up. Once we *know* that there is exactly one so-and-so, we can freely use the definite description 'the so-and-so' to express propositions that are *about* (in Russell's italicized way) the denotation of the descriptive phrase. If this is a correct reading of Russell, and I think it is, his notion of knowledge by description is a remarkable anticipation of the presuppositional analysis of definite descriptions, according to which the meaning of a sentence containing a definite description, 'the so-and-so', breaks into two parts, one of which is that there is exactly one so-and-so.

The Russellian variation on the presuppositional analysis is to move it from the semantic to the epistemic.[160] To enable *thinking about,* the presupposition must be *known* by the user of the description. When the speaker is expressing knowledge, we may assume that the presupposition is implied by the known proposition in a way that allows us to say that it too is known.[161] But Russell speaks of 'all thinking' and of our succeeding in 'thinking about many things'. He doesn't limit himself to *knowing* about many things. So if in my thinking, I am wont to *query* whether the author of *Waverley* is present at the banquet, the standard Russellian elimination does not correctly capture what Russell tells us about my epistemic state. My query involves two propositions, one queried, which Russell does correctly capture, and another known, which is lost in translation. Note that in 'George IV wished to know whether the author of *Waverley* was present at the banquet' neither choice of scope captures the fact that the query is *about* Scott, and that it involves a bit of knowledge by description. Secondary scope misses the knowledge, and even the mere fact that there *is* exactly one author of *Waverley*, and primary scope makes the query 'by acquaintance'.

[160] It will by now have become clear that I use 'epistemic' and its cognates to cover all the so-called propositional attitudes, not just knowing. However, Russell's two ways for thought to connect with things are epistemic in the narrow sense of requiring knowledge; one requires (direct) knowledge of things and the other requires knowledge of truths.

[161] There are some possible scopey issues that I ignore.

 

Some have criticized Russell's analysis of George IV's query by saying that George IV did not wish to know whether there was exactly one author of *Waverley* because he already knew that. There are two problems with this criticism. First, Russell's analysis of the query does *not* imply that George IV wished to know whether there was exactly one author of *Waverley*. Diogenes wished to know whether there were honest men. This does not imply that he wished to know whether there were men. Second, the criticism gets things backward. The fact that Russell's analysis does *not* imply that George IV knew that there was exactly one author of *Waverley is* a criticism of the analysis, and shows that the mere elimination of the description from the sentential complement to the propositional attitude verb (when the attitude does not imply *knowledge*) does not correctly capture the epistemic situation *as Russell took it to be*.

Here we may hark back to our query whether, according to Russell, one who used a definite description in expressing a thought and one who used the result of Russell's elimination of the definite description to express their thought would express the same objects of knowledge. The answer may be 'yes' for knowledge, because knowledge is a special case, but for thought in general, the answer may be 'no'.

In summary, for Russell, all *thinking about* requires *knowledge*, knowledge by description. If the use of a definite description is to indicate that we know that there is such an individual (which is something like what modern presuppositionalists say), then the mere phrase would signal the presence of this form of *knowledge,* even though the *primary* explicit propositional attitude might be *wishes*, *wonders whether*, *doubts*, etc. In general and in giving examples, Russell neglects to mention this.[162]

The same considerations should apply to my generalization of Russell's knowledge by description. The person who expresses his query by asking, 'Is a Canadian one of the assassins among us?' and the person who expresses her query by asking, 'Is there a Canadian assassin among us?' should be seen, according to the generalized Russell, as being in different epistemic states, the first expressing knowledge that there are assassins among us (along with the wish to know whether one of them is Canadian), the second making no such knowledge claim.

[162] If I am correct in thinking that for Russell the mere phrase should signal the presence of *thinking about*, then the 'very thin notion of *denoting*' in OD may not be so thin after all (though Russell seems never to have noticed this).

 

### *2.2.7 Semantics versus Metaphysics and Epistemology*

Russell's philosophy of language and his epistemology did not mix smoothly. Russell's semantics (unlike Frege's) took the elements of language to represent worldly things.[163] So Russell's propositions, with their already worldly constituents, were ready-made for truth values. Russell needs no additional link between senses and worldly properties or objects (as Frege does, or so I have argued) to connect his propositional constituents to the world.

To Frege's complaint that a huge material object like Mont Blanc with its snowfields could not be a constituent of a thought, Russell had famously replied:

> I believe that in spite of *all* its snowfields Mont Blanc itself is a component part of what is actually asserted in the proposition "Mont Blanc is more than 4000 metres high". We do not assert the thought, for this *is* a private psychological matter: we assert the object of the thought, and this is, to my mind, a certain complex (an objective proposition, one might say) in which Mont Blanc is itself a component part. If we do not admit this, then we get the conclusion that we know nothing at all about Mont Blanc.[164]

This is what his semantics told him. The sentence 'Mont Blanc is more than 4000 metres high', expresses a proposition in which Mont Blanc is itself a constituent.

But Russell also had epistemological commitments as to what could be an object of thought, and these commitments were independent of his semantics. He had his principle of acquaintance: that it is necessary, for the understanding of a proposition, to have *acquaintance* with each of its constituents. This provided an epistemological explanation of how Mont Blanc can be a constituent of an (object of) thought that was separate from and independent of his semantics for names. There is a

[163] He is explicit about properties and relations being worldly in the 'Introduction' to the second edition of *PM*, 'Logic does not know whether there are *n*-adic relations (in intension); this is an empirical question.' This, of course, was written at a much later time (in 1925), but I have no reason to think that his view of our knowledge of properties and relations was not the same in 1902. (Contrast this with Frege's innateness view.) Regarding names, Russell was certainly a Millian, perhaps more extremely so than Mill, and should not be associated with Frege's view of names simply because he claimed that in certain epistemic situations we cannot give a name the direct use 'it always wishes to have' (see below). In 1903 he writes, 'A proper name, such as *Arthur Balfour,* is destitute of *meaning*, but *denotes* an individual', and as late as 1957 he was still proclaiming that '[names] are words which are only significant because there is something [some object] that they mean [name], and if there were not this something they would be empty noises'. Russell's is probably a more extreme view than that of Mill or any contemporary 'direct reference' theorist, all of whom would at least give empty names, like Russell's 'Apollo', a syntactical role, rather then treat them as 'empty noises'. The first quote form Russell is from 'On the Meaning and Denotation of Phrases' in *CP4*; the second is from Russell (1959).

[164] Letter from Russell to Frege dated December 12, 1904. From Frege's *Correspondence*.

 

range of worldly things that experience has brought us into contact with in such a way that we can be said to be *acquainted* with them. These things, though worldly, are also suitable objects of thought. They are the things that we have had 'a presentation of'. In the case of a material object this seems to suggest a fairly close form of causal interaction, with perception as a paradigm. It is an immediate consequence, and one that Russell obviously accepted, that differently situated thinkers would differ in the range of individuals with which they were acquainted, and thus would differ in the range of propositions they could entertain. This is in contrast with Frege's view that the repertoire of thoughts is common to all mankind.[165]

The worldliness of both Russell's propositions and his explanation of *acquaintance* led to two problems, one metaphysical and one epistemological.

The metaphysical problem was that some names, for example, those from mythology, do not, Russell thought, name any individual, although sentences containing these names seem to have meanings. The sentences seem to express propositions, and even propositions that we understand (i.e. can entertain).[166]

The epistemic problem is that there are names in our vocabularies, for example, names of the ancients, which, although they do name individuals, name individuals with whom, according to Russell, we are *not* acquainted. So according to the principle of acquaintance, we should not be able to understand those sentences either, yet we do.

By 1903 he was already aware that his semantics for names was problematic in the case of names from mythology. Russell's way out of this was to make an exception. Names from mythology are not genuine names, but rather abbreviated definite descriptions.[167] But in 1903 he showed no awareness of the problem that proper names of the ancients named individuals with whom we are not acquainted. In the following, note that the worry about 'Apollo' lacking a denotation is not matched by a worry about our being unacquainted with Aeschylus.[168]

[165] The view is expressed early in S&B.

[166] One might have thought that the natural Russellian result, the result most consonant with his semantics, would be to allow a proposition to have a gap in it. There are many difficulties with such a proposal, but Russell seems never to have considered it. The closest he came was the rhetorical question in OD, 'How can a non-entity be the subject of a proposition?' Perhaps he ruled it out because he thought there were proper and improper uses of empty names (but see below). Theories of gappy propositions are explored in Caplan (2002).

[167] He thus anticipates the method that he would use eight years later in KA&KD for names of those with whom we are not acquainted.

[168] Either Russell compartmentalized his semantic and epistemological views from 1903 to 1910,

 

> A proper name, such as *Arthur Balfour*, is destitute of *meaning*, but *denotes* an individual. … We decided that proper names—of which *Apollo* appears to be one—have only denotation, not meaning; but in the present case, the name denotes nothing, since Apollo is a figment. It would seem to result that any phrase in which his name occurs must denote nothing; also, if *Apollo* neither denotes nor means, it becomes a meaningless noise … There is, however, plainly a proper and an improper use of the word *Apollo,* from which it follows that, since nothing is denoted by it, something must be meant. This is, in fact, a general principle with imaginary persons or events; they have not, like actual ones, a definition as *just this*, but they are described by means of a collection of characteristics, of the combination of which they are conceived to be the only instance. Thus when we look up *Apollo* (if we ever do) in a classical dictionary, we find a description which is really a definition; but when we look up (say) Aeschylus, we find a number of statements of which no single one is merely definition, for Aeschylus was who he was, and *every* statement about him is not tautologous [i.e., *no* statement about him is tautologous]. Thus *Apollo* is not a proper name like Aeschylus … Thus imaginary proper names are really substitutes for descriptions.[169, 170]

It is hard to see how the introduction of a definite description drawn from the classical dictionary would advance the project of discriminating proper and improper uses of the name 'Apollo'. (I assume that Russell means that there are truths and falsehoods involving the name 'Apollo' and not merely that it is not an empty noise.) Russell suggests in OD that 'Apollo' means *the sun-god*. But 'Apollo is the sun-god', which one might have thought a proper use of 'Apollo', comes out to be false

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or, in his earlier work, he often pretended that we were acquainted with things we were not, such as the ancients, in order to better explain a different point. I favour the former hypothesis.

[169] 'On the Meaning and Denotation of Phrases' in *CP4.* Similar remarks appear in papers that postdate 'Points About Denoting', in which he introduces the epistemological principle that we must be acquainted with all the elements of any proposition we entertain. Certainly the fact that 'Apollo' abbreviates the description in a classical dictionary keeps popping up, e.g. in 'The Existential Import of Propositions' (1905) and in OD itself. Russell also spoke freely of Socrates and Plato as constituents of propositions, but the context was not always one in which the propositions are objects of thought. The question of context is important here because the *existence* of propositions with the ancients as constituents never comes into question. The issue is only whether such propositions can be objects of thought for *us*. The principle of acquaintance seems to imply a negative answer to that question. However, in 'The Nature of Truth' (1905) pp. 494–5 he speaks about his *belief* and his *assertion* that Caesar crossed the Rubicon, and argues that the objects of such beliefs and assertions are propositions, 'objective complexes' of which external objects are constituents. 'The Nature of Truth' is in *CP4.*

[170] I cannot resist noting that the name 'Arthur Balfour', though lacking in meaning in the technical sense, was meaning laden for Russell. Originally a philosopher, Balfour was then Prime Minister. Fifteen years later, during World War I, which Russell opposed as an absolute pacifist, Russell was sent to prison for having 'in a printed publication made certain statements likely to prejudice His Majesty's relations with the United States of America'. The case turned on a single sentence,

 

both in OD and in Russell's pre-OD theory.[171] It seems that the substitution of a description for the name will only be helpful in securing the intuitive truths if we allow such descriptions to denote a mythical character, perhaps of the kind suggested by Saul Kripke. But then, why not let the name *name* that character (as also suggested by Saul Kripke)?

Whatever the defect of his treatment of 'Apollo' in this 1903 unpublished article, Russell argues in a surprisingly modern way against the idea that a name of an actual individual could abbreviate a description. His remark—when we look up 'Aeschylus' nothing found in a classical dictionary is tautological—anticipates the idea that what is found there is all contingent, and thus cannot be taken to *define* or give the meaning of the name 'Aeschylus'.

By 1910,[172] he does recognize the conflict and chooses to abandon the idea that the names of ancients, *in our mouths*, remain proper names.[173] The names of the ancients get the same substitutes-for-descriptions treatment that 'Apollo' got in 1903.

Russell's suggestion that the description of Julius Caesar that we have before our minds may be merely 'the man whose name was *Julius Caesar*' has perplexed modern readers of KA&KD.[174] But remember, Russell was not making a semantic proposal; he was not trying to explain the relation between a name and what it names. He already had a semantics in which *naming* was a primitive. He was just looking for a good candidate for the description in our minds. There *is* the small problem that many men may have been named 'Julius Caesar', so Russell's semantics

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which Russell later said that he should have put 'in such a way as to prevent misunderstanding by a public not used to the tone of exasperated and pugnacious pacifists'. He was sentenced to six months' imprisonment in the 'Second Division', a somewhat severe form of prison accommodation that increasingly worried Russell. On appeal, his sentence was amended to the First Division. Russell stated that it was Arthur Balfour's intervention (Balfour was then Foreign Secretary) that put him in First Division. It was said that the rules affecting First Division inmates 'have a class flavour about them and are evidently intended to apply to persons of some means who are in the habit of keeping servants.' In the First Division, Russell was allowed the service of another prisoner to relieve him 'from the performance of unaccustomed tasks or offices'. It was there that he finished *Introduction to Mathematical Philosophy*. While Foreign Secretary, Balfour also wrote the Balfour Declaration of 'a recognition of the right to a Jewish state in Palestine'. See Clark (1975).

[171] And Russell seems to know it. See OD p. 491. where he notes that because there is no sun-god, every sentence in which 'Apollo' has primary scope will be false.

[172] In KA&KD.

[173] His example is 'Julius Caesar' (see below), but 'Bismarck' gets the same treatment because he has coincidentally adopted an epistemology that rules out all acquaintance with external material objects.

[174] Russell was aware of the difference between this and 'the man called *Julius Caesar*'.

 

needs to be supplemented by an appropriate theory of the individuation of names.[175] With such a supplement, the idea seems superior to most other suggestions. It is universal in two ways: it applies to all names and it applies to all speakers.[176] It also seems to be the form of description that holds the most promise of closing the gap between our lexicons and what we can have knowledge *about*. Some modal tweaking may be required to make 'Aeschylus is the man named "Aeschylus"' tautological, but even that doesn't seem an immediate impossibility.[177, 178, 179]

Russell ends up with *two* simultaneous views about the 'meaning' of a proper name, that is, two views about what the name contributes to the proposition. One tells us what the name means *simpliciter*; it means (and contributes) its bearer. The other tells us what it means in the mouth (or mind) of anyone who is not acquainted with its bearer; it contributes whatever a particular definite description contributes. Frege, who took the elements of language to represent elements of cognition in the first place never had to face Russell's problem of bifurcating meanings. But Russell stayed pretty firm that the representational function of language was to represent the world, not our thought about the world.

That *we* can't mean what our words mean is puzzling at best. Russell retained a residual ambivalence about allowing the demands of his epistemology to override his semantic convictions. In 1912, he plainly still felt the pull of his 1903 insight that 'Aeschylus was who he was, and *every* statement about him is not tautologous', as is evidenced by these remarks:

> Assuming that there is such a thing as direct acquaintance with oneself, Bismarck himself might have used his name directly to designate the particular

[175] One possibility is the view in my 'Words' (1990).

[176] Must the speaker have the concept of *naming* for the epistemologist to attribute the proposition expressed using such a definite description? Russell does say we must be acquainted with all the properties and relations that occur in the objects of our thoughts. But he is also pretty liberal in allowing such acquaintance, and he doesn't discuss the examples much. In any case, one might claim that anyone who can *use* a name must be aware, at some level, that things *have* names. 'What's your name?' becomes answerable at a pretty young age, though it does require that 'name' be in the child's lexicon.

[177] Russell slightly misspoke in saying 'the man'. This phrase probably has to be replaced by 'the individual' to make it tautological.

[178] An ancestor, though probably not the origin, of this idea of Russell is, of course, Frege's claim in his *Begriffsschrift* that identity sentences are about the names. I believe that Frege too, at the time, relied on a semantics in which naming is primitive. The theory becomes incoherent when we try to form a description from properties associated with what is an abbreviation of a description formed from such properties in the first place.

[179] A version of these ideas is developed in Rickless (1996).

 

> person with whom he was acquainted. In this case, if he made a judgement about himself, he himself might be a constituent of the judgement. Here the proper name has the direct use which it always wishes to have, as simply standing for a certain object, not for a description of the object … It would seem that, when we make a statement about something only known by description, we often *intend* to make our statement, not in the form involving the description, but about the actual thing described. That is to say, when we say anything about Bismarck [he means, when we, who are not acquainted with Bismarck, use the name 'Bismarck'], we should like, if we could, to make the judgement which Bismarck alone can make, namely, the judgement of which he himself is a constituent. In this we are necessarily defeated, since the actual Bismarck is unknown to us. But we know that there is an object B called Bismarck [he means, called 'Bismarck'], and that B was an astute diplomatist. We can thus *describe* the proposition we should like to affirm, namely "B was an astute diplomatist", where B is the object that was Bismarck.[180, 181] [Underlining added.]

Although this passage comes after 1910, and depends on the sense-data theory (that only Bismarck is acquainted with Bismarck), the thrust of what is said could just as well have been said in 1903, substituting 'the judgment which only those acquainted with Bismarck can make' for 'the judgment which Bismarck alone can make'. The fact that Russell has now concluded that only Bismarck is acquainted with Bismarck is irrelevant to the point being made.

It is a coincidence that in the same paper in which Russell notices that we are not acquainted with the ancients, he also abandons the idea that we are acquainted with *any* external, material objects. This can be confusing because the issues are quite separate. As I read the two KA&KD papers there are two strands of thought. The primary one is to elaborate on the *structure* of his 1903 distinction between knowledge by acquaintance and knowledge by description without prejudice as to what particulars we are acquainted with. But there is also a secondary strand in which he *assumes* that our knowledge of external material objects is only through our acquaintance with sense-data. Given the

[180] KA&KD. The passage appears in both versions. He says 'judgement'; I say 'judgment'.

[181] Because Russell speaks of the *use* of a name, I take his view to be that names have several uses, and so the very name that Bismarck can use 'directly', Russell must use as an abbreviation of a description. This suggests that if Bismarck were to assert sincerely, 'Bismarck is an acute diplomatist' (in the style of Bob Dole talking about himself), and Russell were to report 'Bismarck believes that Bismarck is an acute diplomatist', Russell would report falsely. So it seems that, in general, a name must express a description when it occurs within a true belief report, though it may simply stand for the thing named when it is not within such a context. This produces additional regrettable consequences. Even without the sense-data theory, the same problem appears in terms of what Aristotle said using the name 'Plato', and what we can report about what Aristotle said through our use of the name 'Plato'.

 

changes that Russell's views were undergoing during this time, it is difficult to pick the strands apart. I would argue that a close reading of the two KA&KD papers suggests that he recognized that even in his old sense of *acquaintance*, when the objects of perception were still external, we were not acquainted with the ancients. For example, he talks of *degrees* of acquaintance, and he takes note of the fact that 'those who only know of [Bismarck] through history' were not acquainted with Bismarck. This is his old theory of what we can be acquainted with.

I conclude that he *should* have realized in 1903 that despite the names' being in his lexicon, he was not, in his then current usage, acquainted with the ancients, and he should have realized that those who only knew of Bismarck through history were not, in his then current usage, acquainted with Bismarck.

It is tempting to imagine Russell writing, in 1903, a passage quite like Frege's famous footnote to S&B (1892) about the name 'Aristotle' abbreviating different definite descriptions for different speakers. In fact, it took to 1910 before Russell did write it (about Julius Caesar and Bismarck), but with his own twist.

> … in order to discover what is actually in my mind when I judge about Julius Caesar, we must substitute for the proper name a description made up of some of the things I know about him. … What enables us to communicate in spite of the varying descriptions we employ is that we know there is a true proposition concerning the actual [individual], and that however we may vary the description (so long as the description is correct) the proposition described is still the same. This proposition, which is described and is known to be true, is what interests us; but we are not acquainted with the proposition itself, and do not know *it* though we know it is true.[182]

Russell and Frege both noted and worried about the communication problem: How is it that we are able to communicate in spite of the varying descriptions we employ? But there is a difference between them. Russell never gave up the idea that a name's only *semantic* value was the thing named, so our understanding of the name as abbreviating a description was the best of bad choices, forced by our epistemic limitations. Russell thus has greater resources available to explain, for example, what it means for the description in one's mind to be *correct*. This intuitive notion—that the description in one's mind is *correct*—has no anchor in Frege's theory.

[182] KA&KD 1910 version. A similar, more extended remark about Julius Caesar is made at the end of the 1912 version. In the above quote, the second part, about the fact that different speakers might have different descriptions in their minds, is actually taken from a passage about Bismarck. The passage is repeated in the 1912 version.

 

The case of names of the ancients is just one symptom of a structural incongruity between the ranges of individuals with whom we are perceptually acquainted and our lexicons of names. Perception is personal, hence local; whereas meaning is conventional, hence communal. Though we inherit most of our lexicon, we must make our own acquaintances. Our meanings owe a debt to our linguistic forebears. But we do not inherit their experiences, only the linguistic fruit. This structural incongruity, stemming from the 1903 principle of acquaintance, was sure to doom Russell's identification of the semantic meaning of the sentence we utter with the content of the associated thought, and it did. But he seemed blissfully unaware of the incongruity until he came upon it for the wrong reason. In 1910, sense-data theory drove him to conclude that a common understanding of the names of the ancients (with whom we are not acquainted) was unlikely and that a common understanding of the names of our acquaintances (with which we *are* perceptually acquainted) was impossible.

### *2.2.8 Must we know what we think?*

The 1903 'Points About Denoting' announces, I believe for the first time, the principle of acquaintance—all thinking has to start from acquaintance—which became a staple of Russell's epistemological thought. For the most part, Russell seems to have taken it as self-evident. It is cited again and again, and is part of the foundation for his later, important paper KA&KD, wherein he makes a first attempt to *justify* the principle.

> The fundamental epistemological principle in the analysis of propositions containing descriptions is this: *Every proposition which we can understand must be composed wholly of constituents with which we are acquainted.* … The chief reason for supposing the principle true is that it seems scarcely possible to believe that we can make a judgement or entertain a supposition without knowing what it is that we are judging or supposing about. If we make a judgement about (say) Julius Caesar, it is plain that the actual person who was Julius Caesar is not a constituent of the judgement.[183]

Now by 1910, Russell had fallen under the baleful influence of sense-data theory, which dictated that we could not be acquainted with any of the material objects of the external world. But I think that there is a very important point here that has, to some extent, been lost in Russell's struggles with the objects of *acquaintance*.

I think that the result that we cannot be acquainted with the material objects of the external world already follows, without the sense-data

[183] KA&KD 1910 version

 

theory, from the justification Russell offers for the principle of acquaintance. And I think that we will have to accept Russell's incredible alternative if we are to retain worldly objects of thought.

Russell wants us not merely to be acquainted with the constituents of the propositions we entertain, he wants us to know *what they are.* This admittedly vague phrase suggests to me something much stronger than mere perceptual acquaintance. Our liability to make *errors of recognition*, say by not recognizing a friend from the back and falsely judging 'That's not Scott', shows that if we *were* acquainted with the individuals of the external world through perception, we would entertain suppositions or make judgments about them without, in a certain sense, knowing *what it is* that we are judging, at least without being able to identify what it is that we are judging. In the case in point, we would not know that our judgment had the form *Scott is not Scott*, and therefore, having affirmed this judgment, we would be immediately prepared to deny it. Hence, we would simultaneously be prepared to affirm and deny the same proposition. If that doesn't show we don't know what we are affirming and denying, I don't know what does.

Many will think that this is not a case of failing to know what one is thinking, or if it is, it is only in a very special sense. Be that as it may, it is puzzling to say that we should not expect to be able to recognize the same thought when it recurs. If we add to the problem of recognizing objects the problem of recognizing worldly properties and relations (which was discussed in connection with Frege's notion of *Bedeutung* and which also afflicts Russell's theory) things become really puzzling. Furthermore, in my view, such a failure to recognize the same thought when it recurs is one of the phenomena that Russell was trying to rule out in saying that when we make a judgment or entertain a supposition we must know what it is that we are judging or supposing about.

This argument, from misrecognition, against acquaintance with external material objects is not an argument that Russell makes. He does say in the *Problems* version of KA&KD, somewhat out of the blue, that acquaintance is characterized by 'perfect and complete' knowledge of the thing, and this does clearly rule out external objects like our friends (and would seem to rule out misrecognition in general). But in context, the remark reads more like a consequence of his assumption of sense-data theory than an argument. Furthermore the claim that acquaintance yields 'perfect and complete' knowledge doesn't accord with an earlier characterization that begins that very paragraph, 'We shall say that we have *acquaintance* with anything of which we are directly aware, without the intermediary of any process of inference or

 

any knowledge of truths'. This characterization does seem like a definition, an attempt to explicate a natural and intuitive distinction. By this characterization, the external objects of perception and the worldly properties and relations should count as objects of acquaintance for all those in a normal mental state whose perceptual faculties have not been damaged by excessive indulgence in philosophy. Leverrier may have *inferred* the existence of Neptune, but we don't *infer* the existence of our friends when we see them.

Let's put *perfect and complete knowledge* of a thing aside, and focus on one of its consequences, *perfect recognition* of a thing. There is an important relation between perfect recognition of a thing and a special kind of knowledge of what we are thinking. Without perfect recognition of the objects of thought, we will never have perfect recognition of what we are thinking, and thus we will always be liable to affirm and deny the same proposition. Let us call the liability to affirm and deny the same proposition *the recognition problem*.[184]

Since perfect recognition *is* a consequence of perfect and complete knowledge, the fact that we don't have the latter with regard to external material objects shows that we don't have the former with respect to external material objects. Now if perfect and complete knowledge of what we are thinking is a requirement of our epistemology, then we have an argument for something like sense-data theory that is entirely independent of arguments from illusion and scepticism about material objects. And this argument doesn't require Russell to start over doing epistemology; it just flows naturally from the reasonable things he had been saying since 1903—once we add in his justificatory premise that it seems scarcely possible that we would not have perfect and complete knowledge of what we are thinking.

If we think of Frege's identity puzzle as involving two uses of a perceptual demonstrative, first pointing to Venus in the morning, and then pointing to Venus in the evening, Frege's puzzle can be seen as a form of the recognition problem. Frege's ultimate solution, that the contents of thought are drawn from the realm of cognitions and not from the realm of external material objects, is not so different from Russell's analysis in terms of sense-data, both moved inward to secure perfect

[184] I first expressed my concern about this issue at the end of section xi of 'Quantifying In' (1969). Although the framework is different, the thrust of the point, put in Russellian language, is that no matter how severely we constrain what will count as *acquaintance* with an external material object, we will not be able to avoid the recognition problem. At the time, I didn't think of this as a form of not knowing what one was thinking. I first came to the latter way of describing the situation in 1971 when I was writing my paper 'Dthat' (1978). The recognition problem is widespread, from Frege's 'Hesperus' and 'Phosphorus' to Kripke's Pierre.

 

recognition, though, as noted, I don't think Russell was motivated by the recognition problem.[185,186,187] Still, I have raised worries as to whether Frege's semantic theory, when it tries to connect back to the world through Bedeutung or some other link, is not itself subject to a form of the recognition problem, the form concerned with the recognition of worldly properties and relations.[188]

I accept Russell's transcendental conditional: *If we cannot make a judgment or entertain a supposition without knowing what it is that we are judging or supposing about, then every proposition which we can understand must be composed wholly of constituents with which we are acquainted and for which we have perfect recognition.* However, I think that we do have non-descriptive knowledge of the external world. The fact that external things are *represented* to us (and to other animals) in perception certainly does not imply that our knowledge of them is *descriptive*, and it surely doesn't require 'the intermediary of any process of inference or knowledge of truths' (unless the 'calculations' of our perceptual systems are suddenly going to count as *our* inferences). But perception is always subject to errors of recognition.[189] So I draw the conclusion that Russell found 'scarcely possible to believe', that, in a certain specifiable sense, we often do not know what it is that we are judging or supposing.

My point here does not depend on the 'know what we are thinking' formulation. Put more directly, I claim that although it is puzzling to say that we should not expect to be able to recognize the same thought when it recurs, and puzzling to say that rationality cannot be expected to protect us from simultaneously affirming and denying the same proposition, these results are the inevitable outcome of allowing worldly objects and properties to be constituents of the content of our thoughts.

[185] Russell went further and brought the objects of perception themselves inward; Frege always distinguished the object of perception from its 'presentation'.

[186] Frege was aware of the recognition problem as early as *Begriffsschrift* (1879), where he tried the aforementioned quick fix by claiming that identity sentences were about the names rather than the things named. This, in spite of the fact that he already had all the ideas required for his theory of sense and denotation (announced nine years later in S&B). See section 8 of *Begriffsschrift* for both the quick fix and the ideas required for his later theory.

[187] Given the analogy, why does Russell's sense-data solution seem so awkward and unsatisfying (even to Russell) while Frege's Third Realm of shared cognitions seems so elegant?

[188] I refer to my original discussion of the recognition of thoughts on p. 938.

[189] Even the perception of words, as Kripke's (1979) *Paderewski* case shows.

 

### *2.2.9 Conclusion about knowledge by description*

In spite of my cavils, I think that Russell's ideas about knowledge by description and knowledge by acquaintance reflect an important insight. In the first years of the century, Russell struggled with the analysis of denoting phrases and especially with the way they could represent things with which we are not acquainted. We need only be able to understand the descriptive predicates to form a representation of a possibly unperceived object. I think this seemed almost magical to Russell, and it is. Russell's *knowledge by description* focused on definite descriptions. But names carry the same power. We need only be able to 'understand' the name to have a representation of the individual named, and to understand names seems to require even less than to understand descriptive predicates. The names of the ancients empower us to represent them, even though all of our descriptive information about them may be subject to severe doubt.

Let us return to our starting point—that language is a system of representation. From that point of view, we see that Russell was concerned with two quite different sets of issues. The first involves questions about what is represented and how it is represented, questions of semantics and syntax. These are questions about the very constitution of the system of representation. The second involves questions about when and how we are able to *make use of* these representations. We do it, so we know we *can* do it. But *how* do we do it?[190] These questions are not about the system of representation itself; they are about who has *access* to it, in thought or in communication. These are post-semantical questions concerning our grasp of systems of representation.

Suppose we take Russell to be roughly correct, as I think he is, regarding what is represented by language and regarding the identification of his *objective propositions* (represented by sentences) with *the objects of thought* or, in modern terms, *the content of a thought*. In many cases we reach the content of a thought *through* language.

But not all linguistic expressions provide us with the means to reach what they represent. Sentences in foreign languages and sentences in our own language that contain words we don't comprehend fail to provide such means. It is in connection with this ordinary and familiar notion of linguistic comprehension that Russell went badly wrong. Russell thought that to *use* language we need a special epistemological relation to what the language represents. He thought that we need to be *acquainted* with all of the constituents of *what is represented* in order to

[190] Other species cannot do it, so we should give thanks to the blessings of evolution for this marvellous and almost magical power.

 

*comprehend* the linguistic representation. This, I think, is wrong. To use language we need a special relation (I hesitate to call it *epistemological*) to the *linguistic representations*. I think that this special relation simply *is* what is commonly called *linguistic comprehension* or *understanding*. And I tend to think that *comprehension* is primarily a matter of one's standing within a linguistic community. This standing may itself confer an indirect connection to what is represented within the language, but it by no means requires *acquaintance* with what is represented nor even *knowledge of* it.

We don't know very much about comprehension, in particular, we don't know what the conditions are in which we should say that we comprehend or understand a sentence. It is not a merely subjective matter. If you refer to Aristotle (the philosopher from Stagira) and I take you to be referring to Aristotle (the shipping magnate from Turkey), I may believe that I have comprehended, but I haven't.[191] In my view and contra Russell, comprehension does not require a prior relationship to what is represented. In the case of historical proper names, our first awareness of the individual named is often through learning the name. I think that comprehension of proper names and natural kind terms is largely independent of what is represented because the very same indefinite, and not necessarily correct, characterization may suffice to induce comprehension of a great many names of distinct individuals and kinds. Since natural kind terms often play the critical role in predicates, this implies that our comprehension of many predicates does not require acquaintance with the property that is represented.

We often describe linguistic comprehension by saying that we 'know what the sentence means'. I reject this idiom in so far as it is thought to give a literal rendering of what it is to comprehend a sentence. This formulation leads us too quickly into Russell's error of thinking that we must be acquainted with what the sentence represents, and this we know is wrong. We can certainly comprehend the sentence, 'Timbuktu was a trading center for myrrh' (it wasn't) without knowing much of anything about Timbuktu or myrrh, and without ever having seen either. We don't have to *know who* Aeschylus is to comprehend his name well enough to talk about him, as is shown by the fact that if we incorrectly identify him as a student of Plato, it doesn't affect our ability to use the name to refer to him.[192]

[191] This is an instance of the recognition problem for words, which I discuss in 'Words'.

[192] It may be important to comprehension to have an idea of who he is, but unimportant whether the idea is right or wrong.

 

This then is my claim: The key to our *use* of language is comprehension of the linguistic *representations*, not acquaintance with that which is *represented*. It is the language that we need to connect with. When we comprehend the representation, we can use it to reach what it represents, *its* content. When we don't, the linguistic representations are inert.

The foregoing distinction between comprehension-conditions and semantic representation leads us to view Frege's puzzle (where names are involved) as belonging not to semantics but to the theory of comprehension. When true, 'A=A' and 'A=B' represent the same complex state. But our comprehension of these two sentences tells us only that the first is true. The puzzle, loosely reformulated, is this: If 'A=A' and 'A=B' represent the same state of affairs, why is our understanding of the two sentences so different? This is a puzzle that an adequate theory of linguistic comprehension should solve. In particular, we need to know what conditions suffice for us to comprehend a name. Frege's puzzle, as I have presented it, becomes a puzzle that is essentially connected with our grasp of artificial systems of representation. Note that this approach (if it works at all) resolves a version of Frege's puzzle *within* Frege's own theory. Given that 'pail' and 'bucket' are synonymous, 'Every pail is a pail.' and 'Every pail is a bucket.' should, on Frege's theory, 'represent' the same thought. But our comprehension of these two sentences tells us (with certainty) only that the first is true.[193] We should be able to avoid the implausible result that distinct words cannot have the same sense by using a theory of comprehension to explain how our understanding of the two sentences can differ.[194]

The puzzle about identity was a centerpiece for Frege. Frege thought that it showed that names don't mean or represent what they refer to, but he was wrong. What it shows is that we can comprehend names without recognizing what they mean or represent. It is simply by comprehending the sentence 'Bismarck was an acute diplomatist', rather than by knowing Bismarck or knowing who he was, that we can, as Russell wanted us to, entertain the thought that Bismarck himself was an acute diplomatist.

One might think that Frege's theory of *senses* could be taken as a theory of linguistic comprehension. It cannot. For Frege, the senses play

[193] This is a version of Mates's (1950) puzzle as well as of the paradox of analysis. The paradox of analysis is posed by Langford (1942) against G. E. Moore and replied to by Moore (1942). Moore says that although the *analysandum* and the *analysans* are the same *concept* in a correct analysis, they have different *expressions* [and so may be comprehended differently?].

[194] I owe this point to Joseph Almog.

 

critical semantic roles: It is the senses that linguistic expressions represent, and it is the senses that determine the denotations of linguistic expressions. In contrast, the theory of linguistic comprehension, as I envisage it, plays no semantic role. It belongs to a layer of general semiotics that lies beyond semantics and that presupposes the results of semantic theory.[195] I believe that reasoning takes place in this layer.

Russell's *principle of acquaintance* restricts each of us to a lexicon of representations of things that we have individually perceived or abstracted from our immediate environment; whereas his introduction of *denoting* and *knowledge by description* extends our epistemic range by applying a determiner-phrase syntax to that restricted lexicon. I believe that language allows us to cast aside the constraints of Russell's individualistic principle of acquaintance and to pool our community's perceptual resources. For me, the most important factors in extending our epistemic range lie outside of both syntax and semantics. Our social natures enhanced by our evolved power to manipulate systems of representation have given us access to a range of objects that creatures locked within their individual perceptual systems cannot hope to attain.[196]

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[195] I am tentatively inclined to believe that the *characters* of my 'Demonstratives' play a dual role; they play a semantic role within the system of representation, and (in some internalized form) they are also explanatory of our *comprehension* of indexicals and demonstratives (taken as individual lexical items). This is an unusually high comprehension requirement, so I may be wrong about this.

[196] I think it fair to say that Russell's semantic theory of names has prevailed, while Russell's epistemic problem about names—how do we understand sentences containing names?—is still with us.

 

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