REFERENCES

Barcan, R. C. 1946. A Functional Calculus of First Order Based on Strict Implication. *Journal of Symbolic Logic* 11: 1–16.
Cresswell, M. J. 1967. The Interpretation of Some Lewis Systems of Modal Logic. *Australasian Journal of Philosophy* 45: 198–206.
1972. The Completeness of S1 and Some Related Systems. *Notre Dame Journal of Formal Logic* 13: 483–95.
Feys, R. 1937. Les logiques nouvelles des modalités. *Revue Néoscholastique de Philosophie* 40: 217–52.
Girle, R. A. 1975. S1 ≠ S0.9 *Notre Dame Journal of Formal Logic* 16: 339–44.
Hughes, G. E., and M. J. Cresswell. 1968. *An Introduction to Modal Logic* (IML). London: Methuen.
1984. *A Companion to Modal Logic*. London: Methuen.
Kripke, S. A. 1965. Semantical Analysis of Modal Logic II, Non-normal Modal Propositional Calculi. In J. W. Addison et al. (eds.), *The Theory of Models*. Amsterdam: North Holland, pp. 206–20.
Lewis, C. I. 1918. *A Survey of Symbolic Logic*. Berkeley: University of California Press (N.B.: The chapter on strict implication is not included in the 1961 Dover reprint.)
Lewis, C. I., and C. H. Langford. 1932. *Symbolic Logic*. New York: Dover Publications 1932, 2nd ed. 1961.
Schotch, P. K. 1984. Remarks on the Semantics of Non-normal Modal Logics. *Topoi* 3: 85–90.
Schotch, P. K., and R. E. Jennings. 1981. Non-Kripkean deontic logic. In R. Hilpinen (ed.), *New Studies in Deontic Logic*. Dordrecht: Reidel, pp. 149–62.
Segerberg, K. 1971. *An Essay in Classical Modal Logic*. Uppsala: Filosofiska Studier.
Sobociński, B. 1953. Note on a System of Feys–von Wright. *Journal of Computing Systems* 1:171–8.
Sylvan, R. 1989. Relational Semantics for All Lewis, Lemmon and Feys' Modal Logics, Most Notably for Systems between S0.3^0 and S1. *Journal of Non-classical Logic* 6: 19–40.
Wright, G. H. von. 1951. *An Essay in Modal Logic*. Amsterdam: North Holland.
Zeman, J. J. 1973. *Modal Logic: The Lewis Modal Systems*. Oxford: Clarendon Press.

4

A Problem in Possible-World Semantics

DAVID KAPLAN

I. Introduction

I believe that there is a problem in the conceptual/mathematical foundation of *possible-world semantics* (PWS) which threatens its use as a correct basis for doing the model theory of intensional languages, not of all intensional languages, but of some.[1] This is *not* the notorious problem that logically equivalent expressions have the same intension in PWS. It is a problem that arises even if we limit our intensional languages to notions for which substitution under logical equivalence is entirely correct.[2] I can't say that I fully understand the problem, though I have thought about it off and on for about fifteen years.

I call the problem conceptual/mathematical. Mathematically, it is comparable to, indeed it uses an argument based on the same mathematical foundation as, Russell's paradox, which displayed a contradiction in naive set theory. I cannot give my problem quite so complete a mathematical formulation, but I do think that it shows a serious difficulty in the naive foundations of PWS.

We may approach the problem by means of an analogy. Suppose that one proposed to conceive of the semantics of first-order quantification theory in terms of the method of quantifier elimination and truth-functional evaluation in successively larger finite domains, the logical truths being thought of as those sentences that transform into tautologies in every case.

In an age of Frege, Ruth Barcan Marcus's Russellian instincts, first expressed in the path-breaking "A Functional Calculus of First Order Based on Strict Implication," and reiterated in an influential symposium with Quine and Kripke, can be seen as opening the door to the flourishing international trade in quantified modalities. Though I was only a late convert to her Russellianism, I was early captivated by her remarkable intellect, scholarly range, and philosophical insight, always accompanied by her sprightly, ever good-humored charm. Ours has been a sustaining friendship for well over two decades. She continues to hold my admiration and great personal affection. ["A Functional Calculus of First Order Based on Strict Implication" is in the *Journal of Symbolic Logic* 11 (1946): 1–16. The symposium includes Marcus's "Modalities and Intensional Languages," *Synthese* 13 (1961), and "Discussion," *Synthese* 14 (1962), both reprinted in Ruth Marcus, *Modalities: Philosophical Essays* (Oxford: Oxford University Press, 1993).]

© Copyright by David Kaplan.

There is nothing incoherent or contradictory about such a method. It is just that because it excludes the infinite models, it does not capture the proper intuitive idea of *all* possibilities, which is the idea we aimed to explicate with our notion of logical truth. The problem with this method does not show up in monadic quantification theory. But it constrains the interpretation of non-logical *dyadic* predicates by making it impossible for there to be a relation satisfying the intuitively consistent sentence:

$$\forall x \exists y Rxy \wedge \forall xyz(Rxy \wedge Ryz \rightarrow Rxz) \wedge \forall x {\sim} Rxx$$

So here, in essence, is the problem in PWS. The possible-worlds methodology imposes constraints on the properties expressed by intensional constants, constraints that make it impossible for there to be properties satisfying certain seemingly benign conditions. It appears, therefore, that PWS does not encompass all the intuitive possibilities. The fact that not all possibilities are included seems to be an unintended consequence of the fundamental ideas we associate with PWS.

II. The Problem

There are difficult and profound philosophical issues regarding intensional notions. For example, some may subscribe to a metaphysics according to which all of the intensional supervenes on the extensional, that what is said, known, believed,[3] required, forbidden, desirable, necessary, or contingent is, in this metaphysical sense, supervenient on the distribution of earth, air, fire, and water. According to this view, once we have represented all the possible distributions of the elements, we have represented all possibilities.[4] Thus, each complete distribution of the elements either would necessitate it being desirable that all should be water or would necessitate it not being desirable that all should be water.

But if PWS is to serve for intensional *logic*, we should not build such metaphysical prejudices into it. We logicians strive to *serve* ideologies not to constrain them.[5] Thus, insofar as possible, our intensional logic should be neutral with respect to such issues. Concretely, this means that our lexicon should permit, in every grammatical category, nonsupervenient, nonlogical constants expressing arbitrary properties. For example, we would ordinarily expect our modal logic to permit the existence of a property such that for any set of individuals it is possible that the property apply to exactly those individuals.[6] As logicians, we would not *require* that there be such a property, for to do so would be to refute Hyperdeterminism – the view that *whatever is, is necessary* – according to which a property *could* not apply to what it *does* not apply to. To require this would be to meddle in metaphysics. But I can think of no plausible reason why logic itself should refute the existence of such a property.[7]

A somewhat weaker property, here expressed by the monadic predicate U, is one such that for any individual, x, it is possible that the property applies to x and only to x.

$$\forall x \Diamond \forall y(Uy \leftrightarrow y = x)$$

It takes ingenuity to think of an example of such a predicate[8] (more ingenuity when the domain of individuals is infinite), but again it seems plain that an adequate modal logic should not rule it out on *logical* grounds.

If we now consider propositions and their properties (as usually expressed by intensional sentential operators), the analog to the monadic predicate U would be a sentential operator, Q, such that for any proposition, p, it is possible that the property expressed by Q holds of p and only of p.

(A) $$\forall p \Diamond \forall q\, (Qp \leftrightarrow p = q)$$[9]

Natural examples of operators, Q, that satisfy (A) are difficult to think of. Perhaps, for every proposition, it is possible that it and only it is *Queried*.[10] Perhaps not.[11] It shouldn't really matter. There may be no operator expressible in English which satisfies (A). Still, *logic* shouldn't rule it out.

In the usual possible worlds methodology,[12] a model for a sentence such as (A) has the following features.

(1) We are given a set W (of "possible worlds").
(2) If $w \in W$, ϕ is a formula, and f an assignment of values to variables, then f satisfies $\Diamond\phi$ in w iff f satisfies ϕ in some $w' \in W$

If the language is extended in the natural way to include propositional variables and nonlogical sentential operators, we add

(3) The propositional variables range over all subsets of W (the 'propositions').
(4) We are given an assignment, I, of intensions[13] to nonlogical constants. For the nonlogical sentential operator Q, $I(Q)$ assigns to each $w \in W$, a set of propositions (i.e., a family of subsets of W).
(5) If $w \in W$, ϕ is a formula, and f an assignment of values to variables, then f satisfies $Q\phi$ in w iff $\{w' \in W: f \text{ satisfies } \phi \text{ in } w'\} \in I(Q)(w)$.[14]

(A) has no models of this kind.[15]

III. Propositions

In PWS, wherein propositions just are (or are represented by) sets of possible worlds, there are plainly more propositions than possible worlds. But even on other natural conceptions of the relation between propositions and possible worlds, it seems odd for there to be a set of possible worlds not characterized by any proposition.

I take it that a (the) fundamental idea of PWS is to take possible worlds seriously, not necessarily by being a 'realist' about possible worlds, but by using possible worlds as a fundamental conceptual building block in the analysis of certain forms of speech.[16] There is an analogous and intimately related theory (or program), which we might call *possible-world metaphysics* (PWM), that attempts to use the same conceptual apparatus to analyze directly certain metaphysical notions (forms of speech aside). In both PWS and PWM it is traditional to introduce propositions as ancillary entities, definable in terms of possible worlds. There are, of course, other ways to proceed. One could, consistent with the possible-worlds apparatus, take the notion of proposition as a separate, primitive idea, and develop a theory about the relation between propositions and possible worlds, including the relation between a proposition and the set of possible worlds at which it holds.[17]

However, so long as the notion of a possible world remains fundamental, and the notion of a proposition is not regarded as utterly language dependent,[18] it is hard to conceive why there should be sets of possible worlds not characterizable by any proposition. Would these be sets of worlds so utterly disparate that they can only cohere by enumeration? So disparate that there is no sentence of any possible language of any possible life-form that characterizes them? So disparate that there is no property, no matter how elaborate or arcane, unique to just those worlds? And even then, why *not* characterize sets of worlds by 'enumeration'? If to each world there corresponds a proposition characterizing exactly it,[19] and if arbitrary (including infinite) disjunctions of propositions themselves form propositions (as they do in PWS), then all sets of possible worlds are characterized by propositions.

IV. Doing the Numbers

If propositions are entities, then our modal logic, in its premetaphysical purity, should accommodate a nonlogical sentential operator satisfying (*A*).[20] (*A*) tells us that there are at least as many 'possibilities' as there are propositions. If each 'possibility' corresponds to at least one possible world, then:

(1) There are at least as many possible worlds as propositions.

If possible worlds are entities, then for a variety of reasons:

(2) There are at least as many propositions as sets of possible worlds.

This cannot be.

V. Two Approaches

Without claiming complete understanding, let me try to explain one aspect of the underlying problem as I see it. We may distinguish those approaches to the model theory of PWS wherein the entities representing the possible worlds are thought of as constructed, for example as models for a given lexicon of nonlogical constants, from those approaches in which they are taken as *given* (as points if you will) and the model theory is done on them 'externally'.

Let us consider the first approach. Assume a simple lexicon of nonlogical, extensional constants and the one nonlogical intensional sentential operator *Q*. Each possible world (i.e., on this approach, each model) must assign an extension to each extensional constant, and a family of propositions to the operator. Hence, according to PWS, the model must assign to the operator a family of sets of models. But suppose that the model is itself a member of one of those sets.[21] How can a model assign a family of sets to a constant, if the model itself is a member of one of those sets? Unfoundedness yawns. On normal set-theoretic assumptions this is impossible.

So suppose that instead we assign to the operator a family of sets of models *of level 0* (i.e., models for just the extensional part of the language). This will avoid the unfoundedness difficulty. However, the operator will never hold of a sentence which contains it. No iteration. The operator will only be defined, so to speak, for sentences in the extensional part of the language. And the models containing this assignment will be models *of level 1*.

Models of level 1 seem to give rise to new possibilities beyond those of level 0. At level 0, there were only the possibilities of, say, different distributions of earth, air, fire, and water. Now there are different distributions of earth, air, fire, and water with it being queried whether all is earth, and there are the same distributions with it not being queried whether all is earth.[22] These new possibilities provide new propositions to query – for example, whether it is queried whether all is earth. The question whether this new proposition is queried is not already answered by the initial assignment of a family of sets of models of level 0 to the operator. It didn't arise at that time. The new propositions are represented by sets of models of level 1. Thus the assignment to the operator required to make it true that one of the new propositions is queried must take place in models of level 2. What we see here is the classification of both models and propositions into orders.

One benefit of the classification of propositions into orders is that it tames an intensional version of the Liar Paradox. Consider the sentence

S $\qquad \forall p\,(Qp \rightarrow \sim p).$

If the proposition expressed by S is the only proposition of which *Q* holds, then it is true iff it is false. Hence, given (A), a contradiction ensues.

Do not blame (A)! It is no more pernicious than the empirical assumption that the sentence engraved on Epimenides' tomb is 'The sentence engraved on Epimenides' tomb is false.' The problem lies not in the fact; it lies in our semantics.[23]

On the present approach, the argument fails because it ignores the hierarchical classification of propositions into orders. The argument requires the propositional variable in S to range over the very proposition expressed by S itself, which is not in accordance with the construction as (partially) described.

This process will never give us the big dividend of possible-world semantics: All iterations of all operators being settled at once. It is the commitment to a single fixed class of propositions, closed under application of intensional operators, that gives PWS its characteristic elegance in the treatment of iteration. It would not be PWS without it. Our hierarchical process cannot achieve that result.[24] The historically minded will remember that it was exactly the iteration principles that set Carnap to work trying to create new semantical methods, which ultimately gave us PWS. I note, as an aside, that Carnap's early work did not suffer from the present difficulty because he did not attempt to take account of nonlogical intensional constants.[25]

According to the second approach, a possible world is thought of not as a model, explicitly carrying limited information for a previously fixed lexicon, but as a point ('Let W be an arbitrary nonempty set') which implicitly carries information for all nonlogical constants. (Or perhaps as a Mundus Rasus on which information for arbitrary nonlogical constants can be writ.)

This approach either solves, or helps to conceal, the difficulty that appears so visible in the hierarchical results of the first approach.[26] This external method – advocated nowadays by almost everyone – is just to *assume* that all the possibilities are already out there, each represented by one of the points we call 'possible worlds'. It is this approach that is presupposed by Kuhn when he writes,

> But the point of possible worlds semantics is to interpret a new lexicon of intensional operators entirely in terms of the possible worlds that could have been stipulated before the new lexicon was added.[27]

That is, we have 'external' functions that tell us what the domain of individuals is for the already 'stipulated' possible world represented by each such 'point', what the assignment of values to each of the nonlogical constants is for each such 'point', etc. Kripke models ϕ built on a structure $\langle G, K, R \rangle$ are naturally taken to be of such a kind.[28]

I believe that the same difficulty, which shows up in the first approach by requiring hierarchy, reappears in this approach through constraints like that which makes (A) contradictory. However, most systems of PWS successfully avoid the difficulty, just as Carnap originally did, by avoiding arbitrary nonlogical constants and variables of intensional types.

VI. Closing

Using a theory known to have problems is problematic, but can be stimulating and useful. Naive set theory is like that. Although one doesn't see how to resolve all of the problems, there are areas in which one feels secure. Even on insecure terrain, we can often make good use of a theory. It may be all we have.

I don't feel secure with any nonlogical – perhaps I should say nonsupervening – intensional notions when they are interpreted by PWS. I especially worry about using it for propositional attitudes (even their E-versions), which are classic cases of nonlogical, arguably nonsupervenient, intensional operators. But I won't give up using PWS because I don't have a good alternative and because the problem is still somewhat mysterious.

I have tried to show that naive possible-world semantics leads to a kind of paradox just as naive set theory does, and by means of a similar argument. I also suspect that the ultimate lesson is somewhat the same, namely that the fundamental entities must be arranged in a never completed hierarchy and cannot be taken to be given all at once.[29] The foregoing considerations seem to suggest that an intensional logic that is suited to serve as a general foundation for the introduction of arbitrary nonlogical constants should ramify the intensional types and, in particular, the type of propositions. This was Russell's view. Few espouse it today.

Appendix: Logically Possible versus Metaphysically Possible Worlds

There is a second problem in PWS, as sometimes practiced, to which I wish briefly to call attention. It concerns the attempt to use a single set of intensional notions to simultaneously represent metaphysical and semantical ideas.

Consider, for example, the question of how many distinct, true propositions there are. Church takes up this question and resolves that *qua logician* he should not stipulate to there being more than one true proposition because, I believe, he thought that to do so would be to (negatively) prejudge metaphysical views such as that which I have called Hyperdeterminism.[30] But he also wishes to use his intensional logic to interpret the semantical theory of sense and denotation. Now if the propositions represent (potential) meanings, then Hyperdeterministic metaphysics should have no effect whatsoever on the *number* of propositions. Hyperdeterminism is a *metaphysical* thesis, not a thesis about meaning.[31] The correct Hyperdeterministic conclusion is that all of the (many) true propositions are necessary.

I do not deny that the metaphysical and the logical are interwoven. The surprising result[32] that the logically indeterminate sentence 'Hesperus is Phosphorus' expresses either a necessary or impossible proposition, violated the metaphysical innocence of logicians. But innocence is a relative thing. The metaphysical should not be confused with the logical. If Hyperdeterminism were to imply that there is only one true proposition, then not only would whatever is true be necessary (as would be expected), but *all* propositional operators would become truth functional. Even if this were the only metaphysically possible world, should 'it is desirable that' and 'it is undesirable that' become truth functional?[33] I think not!

A proper PWS framework for a language containing both *possibility* and *desirability* operators should, I believe, allow the logical to dominate the metaphysical. Not the reverse, as Church would have had it. This means that Hyperdeterminism or not, we must retain all the points (representing so-called possible worlds) needed to distinguish the propositions expressed by inequivalent sentences. However, the Hyperdeterminist must regard all but one of these points as representing what is not really possible, in a word, as representing unreal possible worlds. What then does the Hyperdeterminist who is also a possible-worlds Realist say about such points? At this late date, he is surely too sophisticated to fall into the trap leading to the conclusion that desirability becomes truth functional. And he has too much integrity to insist that all logically possible worlds are real even if they aren't really possible. It would be reasonable to take the view that real possible worlds correspond to some of the points, namely, those that are metaphysically possible. But this is not a Realist interpretation of PWS, it is only a Realist interpretation of *real* possibility.[34]

NOTES

1. An abstract of the argument appeared in the *Abstracts of Sections 5 and 12: 7th International Congress of Logic Methodology and Philosophy of Science*. The talk took place during the Congress in Salzburg, Austria, July 11–16, 1983. A more detailed version, on which the present paper is based, was delivered at the Nobel Symposium in Stockholm, Sweden, during the summer of 1986. The first airing of the argument was during the middle 70s. I also presented it as part of my John Locke lectures in Oxford during spring of 1980. The present version has benefited from the quick-response team of Kit Fine and Terence Parsons, who suggested many valuable improvements. The argument has been reported, and sometimes discussed, by M. J. Cresswell [''Quotational Theories of Propositional Attitudes,'' *Journal of Philosophical Logic* (1980); reprinted with updated references in M. J. Cresswell, *Semantical Essays: Possible Worlds and Their Rivals* (Dordrecht, Boston, and London: Kluwer Academic Publishers, 1988): 100–1], Martin Davies [*Meaning, Quantification, Necessity: Themes in Philosophical Logic* (London: Routledge & Kegan Paul, 1981): appendix 9], David Lewis [*On the Plurality of Worlds* (London and New York: Basil Blackwell, 1986): sec. 2.3 ''More Worlds Than There Are?'' pp. 104–8], and Michael Jubien [''Problems with Possible Worlds'' in David F. Austin (ed.), *Philosophical Analysis* (Norwell, Mass.: Kluwer, 1988): 299–322].

2. To notions for which substitution under logical equivalence is not correct, such as 'she whispered that', we may associate an E-analog, 'she E-whispered that', obtained by closing the notion in question under logical equivalence. E-whispering is a rather exotic notion, not often encountered in practical life. Theoretically, however, it is undefective.
3. More property, E-said, E-known, or E-believed.
4. Vague as this is, it already raises delicate questions about whether it is the distribution of *particular* quantities of the elements (such as the very air that, in the actual world, currently fills my lungs) or simply a question of what amounts of what elements are where.
5. Except to valid argument, of course.
6. For example, *being headed for Heaven*, under the old free-will scenario.
7. Hyperdeterminism is my favorite among totally implausible metaphysical views. Ours is the best of all possible worlds because it is the *only* possible world. It is said that it was the rigors of Hyperdeterministic Puritanism from which the original settlers of Southern California were fleeing.
8. Perhaps any one of us could have been the sole winner of the superjackpot.
9. We are unused to seeing propositional expressions flanking '=', although we certainly understand the meaning when the propositional expressions are variables (assuming we understand the domain over which propositional variables range, as PWS assures us we do). In a standard PWS model for an S5 modal logic, in which accessibility is the universal relation among possible worlds, '$q = p$' can be expressed by '$\Box\ (q \leftrightarrow p)$'.
10. That is, it is asked whether it is the case that p. (More properly, it is E-asked.) Note that on this reading, Q is neither closed under consequence nor closed under implication. 'Q (You saw my proof before you published yours)' implies neither 'Q (You published your proof)' nor 'Q (I showed you my proof before you published yours)'. If one insisted that $\Box\ (Q\phi \leftrightarrow Q \sim \phi)$, which I do not believe, it would not affect the substance of what follows, though (A) would take a slightly different form.
11. Here is an argument against (*A*) when Q is so interpreted. For a proposition to be queried, it must be expressed. So if not all propositions can be expressed, then not all can be queried. But it is natural to think that there are more propositions than can be expressed in any one language. For one reason, because it is natural to think that there are only a denumerable number of expressions in any one language, and it is reasonable to think that there are more propositions than that. If we were to accept infinite disjunctions of propositions as forming new propositions (as I am inclined to do, and as PWS does), then for any collection of natural numbers, there would be a proposition saying that my lucky number is among them. This already gives us more than a denumerable number of propositions.

   In defense of (*A*): For it to be *possible* that a given proposition is queried, there need not be some one, previously fixed, language in which it and all other propositions are expressible. On this score, for (*A*) to hold, it need only be the case that for each proposition, it be possible that it is expressed in *some* language, perhaps a different possible language for each proposition. Are there propositions that are not expressible in *any* possible language used by *any* possible beings?

This would be an interesting result. Could it be a result of *logic*?

It should also be recalled that the presence of indexicals in a language can, on some theories, generate a multitude of propositions from just a few words.

12. For a simple S5 modality.
13. In Carnap's sense.
14. A more familiar form for clauses (4) and (5) is to replace $I(Q)$ in clause (4) with an 'accessibility' relation, $\boldsymbol{R}$, between possible worlds, and to replace the right-hand side of clause (5) with a definition in terms of $\boldsymbol{R}$. Clause (5) might then take the form, f satisfies $Q\phi$ in w iff f satisfies ϕ in some [all] w' such that $w\boldsymbol{R}w'$. It is apparent that such clauses can be replaced by clauses of the form of (4) and (5). For example, for the preceding case[s] $I(Q)$ would be defined as follows: If $\mathbf{P} \subseteq W$ (i.e., $\mathbf{P}$ is a proposition), then for all possible worlds w, $\mathbf{P} \in I(Q)(w)$ iff for some w' such that $w\boldsymbol{R}w'$, $w' \in \mathbf{P}$ [for all w' such that $w\boldsymbol{R}w'$, $w' \in \mathbf{P}$].

    Note however that not every intensional operator that can be represented in the form of clauses (4) and (5) can be represented in the more familiar form of a definition in term of an accessibility relation. If there are n worlds, then there are $2^{(n^2)}$ accessibility relations. There are only a denumerable number of definitions, so there are $\omega \times 2^{(n^2)}$ assignments that can be represented in the traditional way. But there are $n \times 2^{(2^n)}$ assignments that can be represented in the form of clauses (4) and (5). The latter is larger than the former when n is infinite.
15. The proof, which I will not spell out here, having already overdosed on cardinality arguments, is the obvious application of Cantor's Theorem.
16. An alternative, not implausible, way to regard the formal model theory associated with PWS is as a "mere" mechanism for articulating and verifying logical relations among sentences of the object language (and perhaps the meaningfulness of certain syntactic forms, such as quantification in), without any commitment to deeper conceptual or metaphysical foundations. On this alternative, the proof of PWS is in the accuracy and the didactic assistance it provides in predicting logical relations among (and perhaps meaningfulness of) object-language sentences containing modal or other intensional expressions. It is these object-language logical relations (and meaningfulness claims) that are subject to independent verification through intuition, philosophical argument, or whatever other means seem appropriate. Such intuitions would concern the notions of *possibility* and *necessity*, not the notion of a *possible world*. This is what I call "*not* taking possible worlds seriously."
17. I rather favor a version of this approach: taking propositions, properties, etc., to be fundamental 'intensional' (in a vague sense) entities, whose exact structure and individuation we do not yet fully understand, and regarding the Carnapian intensions of PWM (in the technical sense of functions from possible worlds to truth values, sets of individuals, and the like) as their 'extensions'. I know this is a confusing use of 'extension'.
18. Those cautious, antiseptic conceptions of proposition that are closely tied to expressibility in some fixed language seem to me to be in odd alliance with the robust, "Get metaphysical!" theology of so many (not all) PWS partisans. Neither the language dependent nor the metaphysical conceptions trouble me on their own, just the odd alliance.
19. Could there be two worlds so utterly alike that they cannot be distinguished except 'numerically'?
20. Suppose K were a collection of possible worlds including all possible distributions of Earth, Air, Fire, and Water. (*A*) requires that there be possible worlds beyond K. Any such world would differ from all members of K with respect to the interpretation of Q, but would duplicate a member of K with respect to the distribution of Earth, Air, Fire, and Water. But this is exactly what Supervenience denies. So Supervenience looks skeptically at (*A*), and ultimately judges it false for *metaphysical reasons*. Fair enough. I have no problem accommodating a metaphysical viewpoint according to which (*A*) is false. I believe that modal logic should accommodate even Hyperdeterminism. But PWS judges (*A*) false for *pre*-metaphysical reasons. Unfair!

21. As it would be if any true proposition had the property expressed by the operator. For example, if the operator were $\Diamond$.
22. The acceptance of both possibilities amounts to a denial of supervenience. As already stressed, our logic should not force the acceptance of metaphysical theses.
23. The foregoing is an alternative argument that PWS, as traditionally practiced, cannot accommodate (*A*). Fine and Parsons both urged presentation of this Cantor-free version of the argument. This version has the additional interest of demonstrating that an initially plausible strategy proposed by David Lewis – "to deny that just any set of worlds gives the content of some possible thought [Query]" (On the Plurality of Worlds, p. 106) – does not, in fact, succeed in rendering (*A*) "unexceptionable" (p. 106) within the PWS framework.
24. There is a natural way in which to construct models at level ω provided that the hierarchy is constructed in such a way as not to redefine Q for propositions for which it has already been defined. There are some technical subtleties in this since the universal proposition, for example, is represented by a different set at level 0 than it is at level 1, but it can be done by requiring that the new assignments *fit together* with earlier assignments in a certain definable way.

    At level ω we will get all iterations without propositional variables settled. But new propositions will still arise. Thus in the case in which propositional variables are involved, as in (A), the difficulty remains.
25. I see the origins of PWS in Carnap's semantical explorations of modal logic in "Modalities and Quantification," *Journal of Symbolic Logic* 11 (1946): 33–64, and *Meaning and Necessity* (Chicago: University of Chicago Press, 1947). In Carnap, the possible worlds are represented by State Descriptions, a variant of our first approach. I believe that the difficulty about iteration presently under discussion does infect Carnap's more general analysis in his "Replies and Systematic Expositions" [in P. Schilpp (ed.), *The Philosophy of Rudolf Carnap* (LaSalle, Ill.: Open Court 1963), pp. 889–905.
26. For those who have the faith (that all the possibilities are really out there, all at once), it solves it.
27. Thomas Kuhn, "Possible Worlds in the History of Science," Preprint (1986), p. 23.
28. Saul Kripke, "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica: Modal and Many Valued Logics* (1963) 83–94.
29. At the turn of the Century, Russell ended *The Principles of Mathematics* (London: Allen & Unwin, 1903) with these words:

    The totality of all logical objects, or of all propositions [or possible worlds?], involves, it would seem, a fundamental logical difficulty. What the complete solution of the difficulty may be, I have not succeeded in discovering; but as it affects the very foundations of reasoning, I earnestly commend the study of it to all students of logic.
30. Alonzo Church, "A Formalization of the Logic of Sense and Denotation," in P. Henle et al. (eds.), *Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer* (New York: Liberal Arts Press, 1951), p. 22.

31. Perhaps this is more apparent if we recall that Church chose, as a partial principle of individuation for propositions what he called "Alternative Two": Logically, equivalent sentences express the same proposition. Is it not clear that Hyperdeterminism should have no effect on whether two sentences are logically equivalent?
32. Due to Saul Kripke.
33. If you doubt that these notions are closed under logical equivalence, use E-desirable and E-undesirable.
34. Readers may think that I am speaking covertly of my respected friend David Lewis, but I am not.

5

Senses of Essence

KIT FINE

One may distinguish between the essential and accidental properties of an object. A property of an object is essential if it must have the property to be what it is; otherwise the property is accidental.

But what exactly is meant by this account? It has been common to give a further explanation in modal terms. A property is taken to be essential when it is necessary that the object have the property or, alternatively, when it is necessary that it have the property if it exist.

For reasons that I have already given in my paper "Essence and Modality," I doubt whether this or any other modal explanation of the notion can succeed. Indeed, I doubt whether there exists any explanation of the notion in fundamentally different terms. But this is not to deny the possibility of further clarification; and it is the aim of the present paper to provide it.

What I shall do is to distinguish some of the closely related ways in which the notion may be understood. This will be important for getting clearer both on which claims can be made with its help and on which concepts can be defined with its help. In particular, we shall see that several different senses of ontological dependence correspond to the different senses of essence. The task is also important for the purpose of developing a logic of essentialist reasoning; for most of the different senses of essence that we distinguish will make a difference to the resulting logic. My main concern in this paper has been with making the distinctions, and not with drawing out their implications; but I hope it is clear from the examples what some of these implications are.

1. Predicational and Sentential Forms

It is helpful to distinguish two different grammatical forms that essentialist claims can assume.

I should like to thank the many people who in numerous discussions have helped me. Ruth Chang read through the whole paper and made valuable comments. My debt to Ruth Barcan Marcus is evident; for she has perhaps done more than anyone, in her technical and philosophical work, to help make the notion of essence respectable. It is therefore with gratitude as well as affection that I dedicate this paper to her.