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SIGNIFICANCE AND ANALYTICITY*

A Comment on Some Recent Proposals of Carnap

ABSTRACT. The most recent, and surely the most notable, attempt to make precise the empiricist thesis:

A sentence is meaningful if and only if it is connected with experience

is that of Carnap in his article, 'The Methodological Use of Theoretical Concepts' [1].

Carnap's approach is to distinguish between observation terms and theoretical terms. He then proposes a method of distinguishing 'significant' theoretical terms from 'non-significant' theoretical terms by means of their connection as given by some theory T with certain observation terms. The present paper reports two consequences of that proposal.

Given almost any theory T, first there is a definitional extension T^* of T such that every theoretical term of T^* (including those of T) is significant (according to Carnap's proposal) with respect to the theory T^*; and secondly there is a 'deoccamization' T^{**} of T such that no theoretical term of T^{**} is significant (according to Carnap's proposal) with respect to the theory T^{**}. The interest in these two results lies in the fact that definitions, though ordinarily thought of as adding no empirical content to a theory, seem to have the power (according to Carnap's proposal) of transforming non-significant terms into significant ones; and the process of deoccamization (which consists of 'splitting' a theoretical term into a conjunction or disjunction of two new theoretical terms) which would ordinarily be thought of as subtracting no empirical content from the theory, seems to have the power (according to Carnap's proposal) of transforming a significant theory into a non-significant one. The possibility of attaining these two results is thought to constitute an inadequacy in Carnap's proposal.

Many empiricists have maintained some version of the thesis:

> A sentence is cognitively meaningful if and only if it is connected with experience.

The connection, however, appears to have weakened with age. When the thesis was first suggested, as part of a program to eradicate metaphysics (or at least that part of it which was considered unconnected), the required connection amounted to an actual *identification* of cognitive meanings with certain experiences. But slogans like, 'the meaning of a sentence *is* its method of verification' have gradually faded to requests that, '(meaningful concepts) be logically connectible with the terms of a suitably chosen observation basis'.

The danger that metaphysics may thus be readmitted to philosophy has been compounded by another tendency in recent formulations of the thesis. They are specified in such a precise way that certain results can be

Jaakko Hintikka (ed.), Rudolf Carnap, Logical Empiricist, 87–94.

shown to be unquestionably entailed by them. Thus, for example, when Ayer proposed that "a statement is meaningful, if some observation statement can be deduced from it in conjunction with certain other premises, without being deducible from those premises alone", Berlin was able to point out that any non-analytic sentence S was thereby meaningful since there is always some observation sentence O, which can be deduced from S in conjunction with $S \supset O$, but cannot be deduced from $S \supset O$ alone.[1] (Note that if S is non-analytic there is some O such that it is not the case that $(\sim S \vdash O)$, but then it is not the case that $(S \supset O \vdash O)$.) Since Ayer's criterion had been so clearly formulated he was forced to either accept that all non-analytic sentences are meaningful or drop the criterion. Ayer, by the way, took the latter course and proposed a new criterion which Church pointed out had the consequence that if there are three independent observation sentences, every sentence or its negation is meaningful. This tendency toward rather precise formulations has put the eradicators in a rather weak position vis à vis the eradicatees (i.e., the metaphysicians). It has been shown that various specifications of the required connection between sentences and experience have entailed that all sentences are meaningful, or that negations of meaningful sentences might be meaningless, or that no universal sentences are meaningful, or (and this has caused the most embarrassment lately) that theoretical physics is meaningless!

A metaphysician who was up on the literature in the philosophy of science might have availed himself of the opportunity to claim that of course his formulations were remote from experience, they are part of the theoretical language. It appears to be extremely difficult to toe that fine line between the electron and the absolute. The most recent and surely the most notable attempt in this direction is that of Carnap in his article, 'The Methodological Use of Theoretical Concepts' which occurs in Volume I of *Minnesota Studies in the Philosophy of Science*. The purpose of this paper is to report certain consequences of that proposal.

Carnap's proposed definition is relative to a kind of language often discussed in investigations into the methodology of science. It is divided into two parts, an observation language L_o and a theoretical language L_T. The observation language contains terms referring to observable properties and relations, etc., and the theoretical language contains terms referring to unobservables. Let V_T be the class of descriptive (i.e., non-

logical) constants of L_T and V_o the class of descriptive constants of L_o. We also consider a finitely axiomatizable theory in L_T and a finite set of connecting postulates. The theory can be represented by the conjunction of its axioms, T, where T is a sentence of L_T; and the connecting postulates can be represented by their conjunction C, which contains terms of both V_T and V_o. The sentences of L_o are usually thought of as completely understood, thus the realm of insignificance is relegated to V_T. Carnap approaches the problem of culling out the significant sentences by way of the elements of V_T which occur in them. He calls a sentence *significant* just in case all elements of V_T which occur in it are significant. The previously mentioned thesis is now read:

> An element of V_T is significant with respect to L_o, L_T, T, and C if and only if it is connected with L_o.

The problem is then to specify the *connection* in terms of L_o, L_T, T, and C.

This is done by means of two definitions:

D1. A term m is *significant relative to the class* K of terms, with respect to L_T, L_o, T and $C =_{\text{DF}}$ the terms of K belong to V_T, m belongs to V_T but not to K, and there are three sentences, S_M and S_K in L_T and S_o in L_o, such that the following conditions are fulfilled:

(a) S_M contains m as its sole descriptive term.
(b) The descriptive terms in S_K belong to K.
(c) The conjunction $S_M . S_K . T . C$ is consistent (i.e., not logically false).
(d) $(S_M . S_K . T . C) \vdash S_o$
(e) Not $((S_K . T . C) \vdash S_o)$

D2. A term m is *significant* with respect to L_T, L_o, T and $C =_{\text{DF}}$ there is a sequence M of terms of V_T, such that the last element of M is m and each element of M is significant relative to the class of those terms which precede it with respect to L_T, L_o, T, and C.

For means of illustration let us study a simple L_o, L_T, V_o, V_T, T, C. $V_o = \{J, P, B\}$, $L_o =$ the class of all sentences of first order logic with identity, containing only elements of V_o as non-logical signs. $V_T = \{b, f, g, h, m, n\}$, $L_T =$ the class of all sentences of first order logic with identity, containing only elements of V_T as non-logical signs.

$$T = (x)(hx \supset fx) . (x)(hx \supset (bx \vee \sim gx)) . (x)(mx \equiv nx),$$
$$C = (x)(Bx \supset hx) . (x)(fx \supset Jx) . (x)(gx \supset Px).$$

It is easily shown from D1 that g is *significant relative to the null class of terms* (taking S_M as $(x)\ gx$, S_K as $(x)\ (x=x)$, and S_o as $(x)\ Px$). Therefore by D2, g is *significant* (taking M as the one term sequence $\langle g \rangle$). A similar procedure establishes that f and h are *significant*. We show that b is *significant relative to* $\{g\}$, by taking S_M as $(\exists x) \sim bx$, S_K as $(x)\ gx$, and S_o as $(\exists x) \sim Bx$. Note, however, that m and n cannot be 'connected' to experience in the sense of the above definitions, since they occur in what Carnap calls an isolated postulate. No sequence of terms beginning with elements of V_o and passing to new terms which occur with some previous term of the sequence in a conjunct of T or C can ever reach m or n. According to Carnap's definition they are not significant, which accords well with our intuition.

Let us call T' a *definitional extension* of T if it is formed from T by the addition of some definitions for new constants in terms of the primitives of T. Let $V_{T'}$ and $L_{T'}$ be the corresponding extensions of V_T and L_T. Note that definitional extensions of a theory are ordinarily thought of as having no more empirical content than the original theory.

I will now show that there is a definitional extension T' of T such that every element of $V_{T'}$ (including m and n) is significant with respect to L_o, $L_{T'}$, T' and C.

Let us form T' by conjoining the following four definitions to T.

Definition $(x)\ (d_1x \equiv (mx.(\exists x)\ (fx)))$.
Definition $(x)\ (d_2x \equiv (mx \supset (\exists x)\ (gx)))$.
Definition $(x)\ (d_3x \equiv (nx.(\exists x)\ (fx)))$.
Definition $(x)\ (d_4x \equiv (nx \supset (\exists x)\ (gx)))$.

It can be immediately seen that d_1 is significant relative to the null class, (taking S_M as $(x)\ d_1x$, S_K as $(x)\ (x=x)$ and S_o as $(\exists x)\ Jx$). Thus d_1 is *significant* with respect to L_o, $L_{T'}$, T' and C. We can next show that d_2 is significant relative to $\{d_1\}$, (taking S_M as $(x)\ d_2x$, S_K as $(\exists x)\ d_1x$, and S_o as $(\exists x)\ Px$). Since we have established that d_1 is significant, d_2 is also. If we now take S_M as $(x)\ mx$, S_K as $(\exists x)\ d_2x$ and S_o as $(\exists x)\ Px$, we can establish that m is significant relative to $\{d_2\}$ and hence, as before, significant.

An analogous procedure would show that d_3, d_4, and n are also significant. This completes our task since the constants of $V_{T'}$ which were significant with respect to L_o, L_T, T and C will obviously also be significant with respect to L_o, $L_{T'}$, T', and C. (The actual details of the above proofs are facilitated by the use of a few well-known theorems about definitional extensions, e.g., that the addition of a definition to a set of premises in primitive notation does not increase the class of derivable sentences in primitive notation.)

Let us call $T'.C'$ a *deoccamization* of $T.C$ if it is formed from $T.C$ either by replacing all occurrences of certain elements of V_T by the conjunction of two new primitive constants of the same type, or by replacing all occurrences of certain elements of V_T by the disjunction of two new primitive constants of the same type. Let $V_{T'}$ and $L_{T'}$ be the result of the corresponding replacements in V_T and L_T. The idea of a deoccamization is that a theoretical term is split into two terms. For example all occurrences of some one place predicate P followed by a single term α, might be replaced by the conjunction $P^1\alpha . P^2\alpha$. Although we would not look with favor upon such a multiplication of entities beyond necessity, I think we would not say that a deoccamization of a theory and its connecting postulates can rob it of empirical content. In fact, any deductive systematization of the sentences of L_o established by $T.C$ is likewise established by any deoccamization of $T.C$. This is easily shown as follows.

Let O_1 and O_2 be two sentences of L_o such that

(1) Not $(\vdash O_1 \supset O_2)$.
(2) $\vdash T.C \supset (O_1 \supset O_2)$.

We then say that $T.C$ establishes $O_1 \supset O_2$. But if $T'.C'$ is a deoccamization of $T.C$, it is a substitution instance of $T.C$. From (2) by substitution we can derive,

(3) $\vdash T'.C' \supset (O_1 \supset O_2)$.

Since the terms being substituted upon do not occur in O_1 or O_2, they (O_1 and O_2) are unaffected. Hence $T'.C'$ establishes $O_1 \supset O_2$.

I will now display a deoccamization $T''.C''$ of a deoccamization $T'.C'$ of $T.C$ such that no element of $V_{T''}$ is significant with respect to L_o, $L_{T''}$, T'' and C''. First,

$V_{T'} = \{b, f^1, f^2, g^1, g^2, h^1, h^2, m, n\}$ $L_{T'}$ = the appropriate class of sentences.

$T' = (x)\ ((h^1x \vee h^2x) \supset (f^1x \vee f^2x))$.
$(x)\ ((h^1x \vee h^2x) \supset (bx \vee \sim(g^1x.g^2x)))$.$(x)\ (mx \equiv nx)$.

$C' = (x)\ (Bx \supset (h^1x \vee h^2x))$.
$(x)\ ((f^1x \vee f^2x) \supset Jx)$.$(x)\ ((g^1x.g^2x) \supset Px)$.

We now deoccamize $T'.C'$ to obtain:

$$V_{T''} = \{b, f^{1,1}, f^{1,2}, f^{2,1}, f^{2,2}, g^1, g^2, h^1, h^2, m, n\},$$
$$L_{T''} = \text{the appropriate class of sentences},$$
$$T'' = (x)((h^1x \vee h^2x) \supset ((f^{1,1}x.f^{1,2}x) \vee (f^{2,1}x.f^{2,2}x))).$$
$$(x)((h^1x \vee h^2x) \supset (bx \vee \sim(g^1x.g^2x)))\,(x)\,(mx \equiv nx).$$
$$C'' = (x)\,(Bx \supset (h^1x \vee h^2x)).$$
$$(x)(((f^{1,1}x.f^{1,2}x) \vee (f^{2,1}x.f^{2,2}x)) \supset Jx).$$
$$(x)((g^1x.g^2x) \supset Px).$$

It is beyond the scope of this paper to offer a proof that no element of $V_{T''}$ is significant with respect to L_o, $L_{T''}$, T'', and C'', but the intuition is quite straightforward. The basic idea is that those terms which were shown to be significant relative to the null class of terms with respect to L_o, L_T, T and C (namely f, g, and h), have been split in such a way that if one attempts to reproduce the original argument with respect to either component, one needs to know that the other component is significant. (The second deoccamization was required by the connection established in the first conjunct of T between h and f.) Thus the sequence of terms referred to in Carnap's D2, can have no beginning.

The former result, that definitions can import significance, suggests that Carnap's proposal may be too weak; whereas the present result, that deoccamization can export significance suggests that it may be too strong.

Let us return to the earlier result. It can be generalized under various sets of hypotheses about L_o, L_T, V_T, T, and C. One of the weakest I have been able to find is the following. The hypothesis says approximately that there are at least two descriptive constants in V_T that lead independently to different observational results. To be more precise:

If L_T, L_o, V_T, T, and C are the appropriate entities, and
(1) $p_1 \in V_T, p_2 \in V_T$, and
(2) there are sentences P_1, P_2, O_1, O_2 such that P_1 contains p_1 as its sole descriptive constant, P_2 contains p_2 as its sole descriptive constant, $O_1 \in L_o$, $O \in L_o$, and
(3) $(P_1.P_2.T.C)$ is consistent
(4) $P_1.T.C \vdash O_1$, Not $[T.C. \vdash O_1]$, and
(5) $P_2.T.C \vdash O_2$, Not $[T.C \vdash O_2]$, and
(6) Not $[P_1.T.C \vdash O_2]$

then there is a theory T' such that T' is a definitional extension of T and every eligible constant of $V_{T'}$ is significant with respect to L_o, $L_{T'}$, T' and C.

I call a term of the theoretical vocabulary eligible if there is some sentence S which contains the term as its sole descriptive constant and such that neither S nor its negation is implied by $T.C$.

Carnap considers in his article a slightly stronger criterion obtained by a modification of D1. It has the result that none of the theoretical constants of our original model L_o, L_T, V_T, V_o, T, C would be considered significant. However, one can still prove a theorem analogous to the above but with a slightly more restrictive hypothesis.

Were it not for the fact that philosophers are licensed to put rather simple observations in seemingly profound settings, one might object that the foregoing merely indicates that Carnap should have restricted the application of his criterion to the *primitive* constants of V_T. This objection appears somewhat stronger when one realizes that Carnap *did* in fact restrict the application of his criterion to the primitive constants of V_T. Even this observation does not seem to me to completely rob the theorem of value, since I, for one, find it of greatest interest to learn that there was a good reason for something I have done. However one can draw further consequences in view of another proposal of Carnap's on a criterion for *analytic* sentences of the theoretical language. This proposal appears in an article entitled, 'Beobachtungssprache und theoretische Sprache', in *Dialectica* for 1958.[2] His proposal has the consequence that if T' has the *form* of a definitional extension of T (where the added constants are now thought of as new *primitives* so that the criterion is clearly applicable), then the sentence $T' \equiv T$ is *analytic* in T'. Again it appears that in a certain sense, T' has no more empirical content than T. Thus it seems inappropriate for the realm of significance to be increased.

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NOTES

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[1] Throughout the body of the paper, all logical signs with the exception of '$\vdash$' are used autonymously. Concatenation is indicated by juxtaposition.

[2] For an English translation, see above pp. 75–85.

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This paper appears exactly as it was written sixteen years ago. It was written to be heard not printed (thus the absence of proofs of the general results), but because of Carnap's reaction to it, reported in [5], I thought the original version might have some curiosity value. At the same meeting at which this paper was presented Carnap began a turn in a different direction with his [6]. But others have pursued refinements of the line herein criticized (see, for example, [7] and a reply in [8], and see [9] and the bibliography therein for refinements of Ayer's original proposal). My objection regarding definitional extensions was independently discovered in 1962 by Wójcicki and is reported in [10]. Rozeboom, whose own criticisms of Carnap's definition appeared in [11], suggested, in a private communication, a simplification of the argument regarding definitional extensions which depended on definitions whose definiens contain both theoretical and observation terms, but such a procedure now seems to me not in the spirit of Carnap's program.

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THE FACTUAL CONTENT OF EMPIRICAL THEORIES

This paper is concerned with the distinction between factual and conventional (analytic) truth. The problem is one of the most controversial one's in the methodology of science, and no solution to it is likely to be commonly accepted. The proposal for dealing with the issue that I am going to examine here is different from the one given by Carnap (cf. [11], [12]); nevertheless is belongs to the same philosophical tradition. I share both Carnap's empiricist attitude and his logical orientation in dealing with problems of philosophy of science.

The first section of the paper is an expository one. It gives a concise presentation of both Carnap's solution to the problem of analyticity and certain further contributions in the field. The second and the third sections are also of a preparatory character. The main ideas of this approach to the problem of factual truth and analyticity that I am going to discuss here will be presented in the fourth section. The fifth contains a brief discussion of the notion of terminological convention. Finally the sixth, which is the last one, shows how Carnap's solution can be reconstructed within the conceptual framework set up in the paper.

1. CARNAP'S PARADIGM OF SOLUTIONS TO THE PROBLEM OF ANALYTICITY

The solutions to the problem of analyticity which have been offered by Carnap and by those who took his approach as a paradigm for their own investigations derive from a more general account of the logical structure of empirical theories. The basic assumptions of this account were established by Carnap in number of writings (cf. especially [10] and [13]). The same or essentially similar assumptions were accepted by a great number of other philosophers, cf. e.g. Hempel [18], [19], Przełęcki [28], and Tuomela [37]. According to this account empirical theories are assumed to be formalized within first order predicate calculus. Each such theory θ is conceived of as the set of logical consequences

Jaakko Hintikka (ed.), Rudolf Carnap, Logical Empiricist, 95–122.

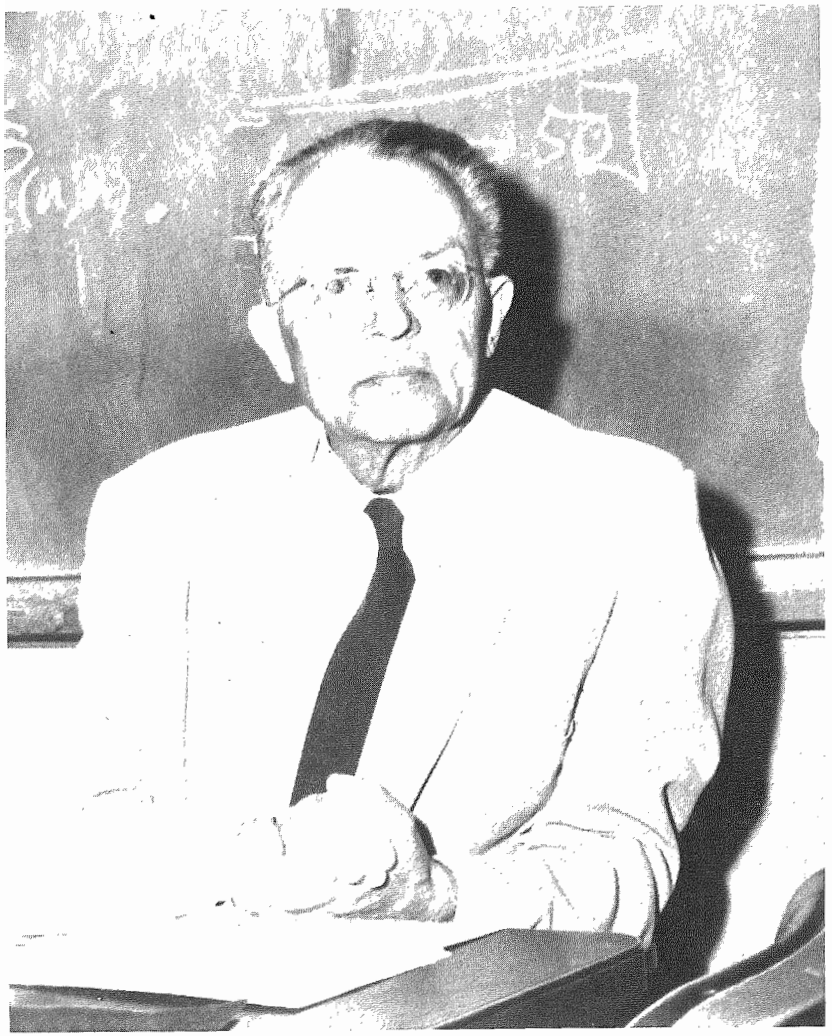

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TABLE OF CONTENTS