the extension of a totally defined formula iff it is hyperarithmetical. The languages $\mathcal{L}_\alpha$ approximating to the minimal fixed point give an interesting "notation-free" version of the hyperarithmetical hierarchy. More generally, if L is the language of an acceptable structure in the sense of Moschovakis, and the Kleene valuation is used, a set is the extension of a monadic formula in the minimal fixed point iff it is inductive in the sense of Moschovakis.[36]

SAUL KRIPKE

Rockefeller University

HOW TO RUSSELL A FREGE-CHURCH *

THE philosophies of language of Frege and Russell are the two great competing classical theories, and any exact comparison of them requires attention to their intensional logics, which represent the pure theoretical (in the sense of theoretical vs. observational) superstructures—or perhaps one should say deep structures—of their theories. My earlier work on the logic of demonstratives, which argued against what I take to be tenets of Frege's philosophy of language, had led me to a greater appreciation of Russell's views. I wanted to determine what essential features of Frege's doctrine could not be accommodated within a Russellian approach. This attempt led to a surprising result.

I

I began by noting that, for a variety of puzzles, including Frege's puzzle about the meaning of identity statements and the three puzzles explicitly discussed by Russell in "On Denoting," one can directly compare the solutions of Frege and Russell and assess the theoretical apparatus each brings into play. (When I refer to Russell's logical doctrines, I have in mind the doctrines of "On Denoting" and the first edition of *Principia Mathematica*. Russell held several other doctrines throughout his career, and, of course, the doctrine of *Principia* was not his alone. In attributing doctrines to Frege, I take account not only of his own writings but of those of his great modern exponent and proponent, Alonzo Church.) De-

[36] Leo Harrington informs me that he has proved the conjecture that a set is the extension of a totally defined monadic formula iff it is hyperelementary. The special case of the Π^1_1 and hyperarithmetical sets if L is number theory is independent of whether the Kleene or the van Fraassen formulation is used. Not so for the general case, where the van Fraassen formulation leads to the Π^1_1 sets rather than the inductive sets.

* To be presented in a joint APA/ASL symposium on Sets, Concepts, and Extensions, December 29, 1975. Charles Parsons will be co-symposiast; his paper is not available at this time.

spite some superficial resemblance—both held something like a disguised definite-description theory of proper names (*most* proper names, for Russell; *all* proper names, and even demonstratives like 'I', for Frege)—the theories are quite different. Frege employs his doctrines of sense and denotation, indirect denotation, and senseful but denotationless expressions. As Church points out, the hierarchy of intensions—senses of expressions denoting individuals, senses of expressions denoting such senses, senses of expressions denoting *such* senses, etc.—seems to be inevitable in such a theory; if not directly from the analysis of iterated operators whose operands have indirect denotation, then by repeating the analysis of contingent identity sentences for entities of higher intensional types. For example, by constructing an identity sentence using descriptions such as 'the sense of Russell's favorite name for Frege' we show the need for senses of expressions denoting senses of expressions denoting individuals.

Russell seems entirely to avoid both the sense/denotation doctrine and the attendant ontological hierarchy by means of his theory of contextually defined incomplete symbols and the consequent doctrine of scope. According to this theory certain expressions, notably definite descriptions, have no meaning in isolation, although each sentence containing such an expression does have a meaning. When the underlying logical form of these sentences, which may be quite different from their apparent grammatical form, is made explicit, the contextually defined expressions completely disappear. Russell's well-known analysis of sentences containing definite descriptions has as an immediate consequence the doctrine that molecular sentences containing definite descriptions are syntactically ambiguous as regards the scope of the definite description.

Thus we see two quite different arrays of conceptual tools, and even distinct ontologies. For Russell, there are individuals, propositions, and, for each n, n-place functions to propositions (such functions are usually called *propositional functions*). For Frege, there are individuals, truth values, for each kind of entity, a second kind capable of being a sense of a name of something of the first kind, and, for each n, n-place functions to entities of *any* given kind. For Frege, sentences are names of truth values. If we take the senses of sentences to be propositions, it would appear that Russell's intensional ontology is a part—and a small part—of Frege's intensional ontology.

We would expect Frege's larger intensional ontology to have some direct utility in the analysis of language, and it appears that we

have found such if we turn to the analyses offered by Frege and Russell of certain so-called "intensional" verbs. There is a general form of invalid but seemingly correct inference which involves interchanging two definite descriptions with a common denotation within the scope of such a verb. The seeming correctness of such an inference is due to the fact that it appears to be an instance of Leibniz's law. Frege blocks such inferences by means of his doctrine of indirect denotation, according to which the two descriptions *in this context* do not have a common denotation. Thus, the premise of Leibniz's law fails (or, if thought of as asserting the identity of the ordinary denotations, is irrelevant). Russell blocks the inference by eliminating the description within the sentential complement of the intensional verb. With the descriptions gone, Leibniz's law is inapplicable and there remains no other source of plausibility for the inference. If the descriptions were eliminated from the whole sentence, rather than just the sentential complement, the inference would be valid (though still not directly by Leibniz's law). This suggests that Russell's solution will fail for intensional verbs that do not take sentential complements, for example, 'seeks'. In "Schliemann sought the site of Troy" there is no *secondary* scope elimination that Russell can use to mimic Frege's analysis. Given, then, that the site of Troy is the site of Burbank, Russell seems unable to block the inference to a false conclusion. But Frege's solution is unaffected. Here we seem to see the value of Frege's additional entities (in the case in question, the senses of names of places).

There is also an important—though less often noted—difference between Frege and Russell regarding the structure of intensional entities. According to Russell, an individual may be an immediate element of a proposition. In fact, certain *atomic propositions* consist of just individuals and attributes (or relations); whereas, for Frege, the immediate elements of a proposition must themselves be intensional entities of one sort or another. There is a direct link between this difference in the two theories and the problem of interpreting quantification across intensional verbs (the kind of quantification that arises when a description which appears to stand within the scope of an intensional verb is regarded as taking the whole sentence as its scope and is eliminated from that context). In our modern-day possible-worlds semantics for intensional languages, the problem of identifying individuals across worlds is yet another manifestation of this same theoretical difference. As I will try to show, this is the crucial difference between Frege and Russell.

II

Most of the above-mentioned features of the intensional doctrines of Frege and Russell—their competing theories of definite descriptions, Frege's larger intensional ontology, and their competing views on the structure of propositions—are well known. What is less well known, or, more accurately, was surprising to me, is the interdependence of these features. I have obtained a result which tends to show that, given Russell's theory of the structure of intensional entities, we can represent all of Frege's ontology within that part of it which constitutes Russell's ontology. Furthermore, the representation is such that the Fregean analysis of sentences containing descriptions within the scope of an intensional verb translates into an approximation to the natural Russellian analysis.

There are several difficulties, both historical and systematic, in my arguments for the above results. From a historical point of view, one problem is that Frege himself gave no formalization of his intensional logic, and Church's recent attempts to do so are not entirely satisfactory for my purposes. Thus I have constructed my own formalization.

A second problem in the historical realm arises from the fact that I give a purely semantical (i.e., model-theoretic) argument for the representation of Frege's ontology within Russell's. This in itself would not be objectionable except for the fact that I have used a possible-worlds form of semantics. This form of semantical theory takes the notions of a possible world and a possible individual in a world as basic, and then uses logical constructions to represent such intensional entities as propositions, individual concepts, properties, etc. But the notions of the structure of intensional entities which underlay Frege's and Russell's proposals were quite different. For Russell, for example, the only basic intensional notion seems to be that of an attribute (or relation in intension). Propositions and propositional functions (which are distinct from attributes) are constructed from these and individuals. Possible worlds might then be represented by maximally consistent sets of propositions, but there is a hitch to the smooth translation between modern possible-worlds semantics and Russellian semantics. Insofar as I have been able to formulate a model-theoretic semantics based on Russell's ideas, what has resulted is a *ramified* theory, which has no absolutely maximal sets of propositions but only maximal sets of propositions of a fixed order.

Another way in which I do not perfectly represent Russell's theory is this. In presenting the Frege-Church ontology I follow

Church in using the Schönfinkel reduction of n-place functions, for $n > 1$, to higher-order 1-place functions. Thus the Russellian sub-ontology to which I reduce the full Frege-Church ontology contains functions whose values are not propositions. These are non-Russellian kinds of entities. This is not a serious defect because it is easy to represent a 1-place function by a 2-place propositional function.

I do not merely reduce the Frege-Church ontology to a Russellian ontology; I also provide an effective set of rules for translating any sentence of the Frege-Church language into one of the Russellian fragment. This translation preserves meaning in the sense of intension. Thus, whatever can be said using the full resources of the ontology can be said with reference only to entities of the Russellian sub-ontology. These sentences, whose ontology is Russellian, are not yet in the language of Russell. They may contain definite descriptions, whereas it is a tenet of Russell's theory that all expressions, and especially definite descriptions, whose denotation is dependent on contingent circumstances must be eliminated. Furthermore, these sentences denote truth values, whereas the sentences in Russell's language denote (or *signify*, to use his term) propositions. However, if we take the natural Fregean symbolization of a sentence of English, form a name of the sense of this sentence (rules are given for transforming an arbitrary expression into a name of its sense), and then apply my translation to this name, the result will be an expression whose ontology is Russellian and which is equivalent to the natural Russellian symbolization of the original sentence of English.

Of course, if the Fregean and Russellian symbolizations produce sentences of differing truth values—as is sometimes claimed for English sentences involving improper definite descriptions—there is no hope for an intension preserving translation which will carry the one into the other. However, from my perspective, the essential difference between Frege's and Russell's treatments of definite descriptions does not lie in these truth-value disparities at all, but rather in how they regard these parts of speech. Thus we could modify the Russellian analysis of sentences containing improper descriptions to make it conform always *in truth value* with Frege's analysis. Or we could modify Frege's analysis of sentences containing improper definite descriptions to make it conform always *in truth value* with Russell's analysis. In neither case would we affect the essential difference between the theories. I have taken the former course and followed Frege-Carnap in regarding 'the' as containing implicit reference to a chosen object. The resulting deviation from Russell, which appears only in the case of an improper description, is only a slight embarrassment. The alternative

of adjusting the Fregean theory to conform with Russell's intuitions regarding truth value would have been more complicated, because I use the chosen objects anyway in the course of the ontological reduction.

III

The Frege-Church *O*ntology—"Churcho" for short—consists of entities of the following non-overlapping types: individuals (type ι); truth values (type o); for any types α, β, functions from entities of type β to entities of type α (type $(\alpha\beta)$); and for any type α, concepts of things of type α (type α_1). It is this last iterative principle that produces the hierarchy of intensions.

We can represent the ontology in the now familiar possible-worlds way as follows. Let W be a non-empty set—thought to represent the set of possible worlds—and let I be a function which assigns to each $w \epsilon W$ a non-empty set—thought to represent the individuals of the possible world represented by w. [To carry through the reduction in detail, we require also a chosen $I^*(w)$ from each of the sets $I(w)$.] For a given $w \epsilon W$ and a given type α, *the universe of the type* α *at* w is represented in the standard way using Carnap's idea that a concept whose type is β_1 (which would be a concept of an entity whose type is β) can be represented by a function which assigns to each possible world representative w, an element of the universe of the type β at w.

The *L*anguage of the Frege-Church ontology—"El Churcho" for short—contains variables of every type, along with logical constants for: the truth-functional conditional, universal quantification, definite descriptions, the lambda operator, the relation *is a concept of*, and the operation of *composition*. Both Churcho and El Churcho are closely modeled on the systems of Church's "A Formulation of the Logic of Sense and Denotation" [1] and "Outline of a Revised Formalization of the Logic of Sense and Denotation." [2] The difference is that Church identifies the type $(\alpha\beta)_1$ with the type $(\alpha_1\beta_1)$. Since I do not, I require the additional notion of *composition* in order to combine a concept of a function with a concept of one of its arguments in order to produce a concept of the function value. Corresponding to each constant of any type α, we require a second of type α_1 to denote the sense of the first, and a third of type α_{1_1} to denote the sense of the second, and so on. Given any closed well-formed expression A, we can effectively find a well-formed expression $\overline{A}$ which denotes the sense of A.

Our possible-worlds representation of Churcho can be converted

[1] In Paul Henle, ed., *Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer* (New York: Liberal Arts, 1951).

[2] *Noûs*, VII, 1 (March 1973): 24–33 (part I); VIII, 2 (May 1973): 135–156 (part II).

into a model for El Churcho by adding a function i which assigns to each constant of type α an *intension* drawn from the universe of α_1. (Since for types of the form α_1, the universes at w and at w' do not differ, we may ignore the relativization to a possible world.) If we let w represent the actual world, a model takes the form $\langle w, W, I, i \rangle$.

IV

When we construct a model of something, we must distinguish those features of the model which represent features of that which we model, from those features which are intrinsic to the model and play no representational role. The latter are *artifacts of the model.* For example, if we use string to make a model of a polygon, the shape of the model represents a feature of the polygon, and the size of the model may or may not represent a feature of the polygon, but the thickness and three-dimensionality of the string is certainly an artifact of the model.

Given any possible-worlds representation of Churcho, constructed from a particular set W and a particular function I, and given any distinct elements w and w' of W, some definite relation, either of overlap or disjointness, will hold between $I(w)$ and $I(w')$. To put it another way, there is no intrinsic problem—within the model—of *identifying* members of $I(w)$ with members of $I(w')$; we need no *criteria* to make precise the question whether a given member of $I(w)$ is also a member of $I(w')$. Thus, the overlaps (or disjointness) between such pairs as $I(w)$ and $I(w')$ is a definite feature of our model. Is it an *artifact of the model* of a *feature of the metaphysical reality* being modeled?

Suppose that we adhere to a metaphysics of possible worlds and possible individuals. Then we probably believe that for each possible world there is a definite number of possible individuals that exist in it. Thus the cardinality of the sets $I(w)$ is not an artifact. But there seems to be some disagreement as to whether we can meaningfully ask whether a possible individual that exists in one possible world also exists in another without taking into account the attributes and behavior of the individuals that exist in the one world and making a comparison with the attributes and behavior of the individuals that exist in the other world. The doctrine that holds that it does make sense to ask—without reference to common attributes and behavior—whether *this* is the same individual in another possible world, that individuals can be extended in logical space (i.e., through possible worlds) in much the way we commonly regard them as being extended in physical space and time, and that a common "thisness" may underlie extreme dissimilarity or distinct

thisnesses may underlie great resemblance, I call *Haecceitism*. (I prefer the pronunciation Hex'-ee-i-tis-m.) It would be more exact to speak of Haecceitism *with respect to* a given kind of entity, but for present purposes we may assume that only individuals are in question and that our individuals are themselves some well-defined kind of entity, perhaps animals.

The opposite view, *Anti-Haecceitism*, holds that for entities of distinct possible worlds there is no notion of trans-world being. They may, of course, be linked by a common concept and distinguished by another concept—as Eisenhower and Nixon are linked across two moments of time by the concept *the president of the United States* and distinguished, at the same pair of moments, by the concept *the most respected member of his party*—but there are, in general, many concepts linking any such pair and many distinguishing them. Each, in his own setting, may be clothed in attributes which cause them to *resemble* one another closely. But there is no metaphysical reality of sameness or difference which underlies the clothes. Our interests may cause us to *identify* individuals of distinct worlds, but we are then creating something—a trans-world continuant—of a kind different from anything given by the metaphysics. Although the Anti-Haecceitist may seem to assert that no possible individual exists in more than one possible world, that view is properly reserved for the Haecceitist who holds to an unusually rigid brand of metaphysical determinism.

Haecceitism holds that we can meaningfully speak of a thing itself—without reference either explicit, implicit, vague, or precise to individuating concepts (other than being *this* thing), defining qualities, essential attributes, or any other of the paraphernalia that enable us to distinguish one thing from another. It may be that each thing has essential attributes with which it is vested at all times and in each possible world in which it exists. But that is an issue posterior to whether things have trans-world being.

If I may re-use the analogy between possible worlds and moments of time, the Haecceitist regrets that we can come to know whether *this* is Anastasia only by a painstaking study and comparison of present attributes with past ones. The Anti-Haecceitist believes there is nothing more to know.

Probably, most of us are Haecceitists with respect to most things through time, but the very inaccessibility of other possible worlds seems to have produced a goodly number of Anti-Haecceitists with respect to trans-world identifications. Even when their quantified modal logics look Haecceitistic, their pre-systematic remarks may explain the so-called identities as a manner of speaking.

The supposition that we adhere to a metaphysics of possible worlds and possible individuals is not inevitable. After all, our primary goal was to model the ontology of the Frege-Church intensional hierarchy. We may, as I believe Church does, regard such remarks as "think of W as representing the set of possible worlds" as of heuristic value but as reflecting no metaphysical commitment. Possible worlds and possible individuals, insofar as it is metaphysically sound to speak of them at all, may be thought of as constructs from such *given* entities of the ontology as propositions, attributes, individual concepts, and the like. This view—that the ontology of Churcho reflects the basic metaphysical commitments—seems more consonant with the outlook of Frege, Russell, and, of course, Church.

I believe that the issue of Haecceitism reappears, within this metaphysical framework, as the question whether an individual itself—as opposed to an individual-under-a-concept—can be an immediate constituent of a proposition. Let us adopt the terminology *singular proposition* for those (purported) propositions which contain individuals as immediate constituents, and *general proposition* for the others. The sentences "All men are mortal" and "The finest man, whosoever he may be, is mortal" are generally conceded to express general propositions. "I am mortal" and "This is blue" are thought by some to express singular propositions.

Why is the acceptance of singular propositions simply another version of Haecceitism? For two reasons: first, if propositions, attributes, etc. are represented in the usual way by functions on possible worlds, then in representing a singular proposition that contains an individual x we would want to assign truth to those possible worlds in which x has whatever property is attributed to him. But this presupposes that, for each world, it is a determinate question which, if any, of its individuals is x. If we are only asked to represent general propositions, we can confine our attention, in each possible world, to considerations *internal* to the life of the world, and the *external* question, "But is it x?" need never arise. The second reason is that if possible worlds are represented as sets of propositions and we permit such singular propositions as that expressed by "I exist" (a true—perhaps even analytic—but contingent proposition), then we have, by way of the individuals that are immediate constituents of such propositions, a metaphysically sound way of identifying certain individuals of one possible world with those of another. On the other hand, if we limit ourselves to general propositions, any such transworld identifications would require a special and independent justification. (There are

complexities here which I will not now attempt to elucidate.) Thus we see that, whichever outlook we take as basic, the acceptance of singular propositions is linked to the acceptance of trans-world identities.

The question of Haecceitism is important to the philosophy of language. I have counted myself a Haecceitist since I saw the issue starkly during my study of the logic of demonstratives. (I have held the position since that time, but it was only recently that Robert Adams suggested the epithet.) If one believes, as I do, that whatever attributes are capitalized upon for the demonstration, demonstratives are devices of pure reference, then one seems committed to accepting something like singular propositions in one's semantics. If one regards the usual form of quantification into modal and other intensional contexts—modality *de re*—as legitimate (without special explanations), then again one seems committed to some form of Haecceitism.

As is well known, there are those who reject the meaningfulness of such forms of speech or suggest other analyses. Frege, for example, seems to reject singular propositions and to believe that all names, even such demonstratives as 'I', are to be analyzed as disguised definite descriptions. Frege is an Anti-Haecceitist. Church, I believe, follows Frege in this aspect of his philosophy.

There can be little doubt that Russell was a Haecceitist. The metaphysics of *Principia Mathematica* (and earlier of *Principles of Mathematics)* gives us atomic propositions with individuals as constituents. Thus "Dion walks" expresses a proposition that contains Dion himself as a constituent (and the attribute *walks* as a component). I believe that, if Russell were willing to adopt the possible-worlds terminology, he would say that such a proposition is represented by that function which assigns truth to a possible world in which the constituent has the attribute and falsehood to one in which he lacks it. I am not sure what he would say about a possible world in which Dion does not exist. Probably he would choose falsehood, not because of his analysis of definite descriptions or disguised definite descriptions—Dion is a genuine constituent of this proposition—but just because his instincts seemed to go that way. (There is, to my knowledge, almost no relevant evidence because Russell did not take modal logic seriously. The situation is further confused by the fact that he regarded 'Dion exists', with 'Dion' a genuine name, as meaningless.[3])

Some may not think of Russell as a Haecceitist because they con-

[3] This point was brought to my attention by Joseph Lambert.

fuse his epistemology with his metaphysics. We *know* (and thus express) very few such singular propositions, according to Russell. The proposition *we* express when we utter "Socrates was wise" does not have Socrates as a constituent. But there *is* such a proposition. We can't know it or even, I suppose, entertain or apprehend it, because we are not acquainted with Socrates. Russell's theory of proper names which are disguised definite descriptions is motivated by epistemological considerations. Although he seems, thus, to explain away singular propositions, in fact they are still required by his metaphysics to construct the general propositions that we do express. I don't fully agree with Russell's epistemology, but the important point is that Haecceitism goes quite smoothly at the metaphysical level, but raises many difficulties at the epistemological level.

Here we are concerned with metaphysics. My aim is to apply Russell's methods in intensional logic—which depend on his Haecceitism—to the Frege-Church hierarchy, hoping to show that Haecceitism is all that stands between Frege-Church and Russell.

Before going on, I wish to call attention to the fact that some may have adopted an Anti-Haecceitist position as a form of *Actualism*. (Again, I am indebted to Robert Adams for the terminology.) We have spoken of possible worlds other than the actual one and possible individuals other than the actual ones. Many, myself included, find such talk ultimately unsatisfactory (though a useful stopgap). They are pleased to say, "It is possible that there is an individual such that so and so" but recoil at "There is a possible individual such that so and so." To be concrete, I can assert that it is possible that there be an individual who is not among the actual individuals (for there might be more individuals than there, in fact, are), without committing myself to the existence of a possible individual who is not actual. It may be feared that Haecceitism must be a two-way street. If it opens the door to let actual individuals into other possible worlds, how can it help but allow what were merely possible individuals from slipping into the actual world? It can. Haecceitism vs. Anti-Haecceitism is a distinct and independent dimension from Actualism vs. Possibilism. All four combinations are consistent. One might claim that Possibilism and Anti-Haecceitism is represented by David Lewis, Possibilism and Haecceitism by Montague, Actualism and Anti-Haecceitism by Church, and Actualism and Haecceitism by myself. I think that Kripke also shares the last position, though he has not spoken explicitly on Actualism vs. Possibilism in print. At any rate, the treatment of quantification in "Semantical Considerations on Modal

Logic"[4] is Actualistic, whether by fancy or determination I do not know. (Note that the remark about 'Sherlock Holmes' denoting a possible individual is withdrawn in the Addenda to "Naming and Necessity."[5]) An Actualist can accept the existence of propositions which are only possibly true. He cannot accept the existence of those possible propositions which, independent of truth value, depend on what is not actual, for example, the additional singular propositions which would exist if there were additional individuals.

V

The Haecceitist will regard overlaps between $I(w)$ and $I(w')$ as representing features of the metaphysical reality; the Anti-Haecceitist will regard them as artifacts of the model.

How can we represent the Anti-Haecceitist's position in our model theory? As was remarked above, not by requiring that $I(w)$ and $I(w')$ be disjoint for $w \neq w'$, since this would equally well represent the metaphysical thesis that no individual exists in more than one possible world, and both this thesis and its negation should count as meaningless according to the Anti-Haecceitist. Those who regard such overlaps as artifacts of the model must "factor out" this feature by defining a notion of *isomorphism* between models which preserves structure except for such overlaps. This is easily done.

Church's position seems to be that models that are isomorphic in this sense represent (i.e., model) the same reality. Thus, he has carefully adopted a form of language for which isomorphic models preserve the truth value of all sentences. (A stronger result holds: given any well-formed expression A of any type and two isomorphic models, the denotation of A in the one model is the image under the isomorphism of the denotation of A in the other model.) Any form of language that violated this would contain elements which, from this viewpoint, were metaphysically meaningless, since the notation would contain sentences whose truth value was determined not by features of reality but by the artificial stipulations required to fix a particular model. Such sentences would permit us to discern the indiscernible!

VI

Returning now to my hypothesis that Frege-Church + Haecceitism = Russell, I will explain the leading idea. For Church, an attribute —the meaning of a predicate—combines with an individual concept to yield a proposition. This suggests that an attribute can be repre-

[4] *Acta Philosophica Fennica,* XVI (1963): 83–94; reprinted in L. Linsky, ed., *Reference and Modality* (New York: Oxford, 1971).

[5] In D. Davidson and G. Harman, eds., *Semantics of Natural Language* (Boston: Reidel, 1972), pp. 764/5.

sented as a function from individual concepts to propositions. (This is the one feature of "A Formulation of the Logic of Sense and Denotation," which I have abandoned in El Churcho.) For Russell, an attribute combines with an individual to yield a (singular) proposition (here the Haecceitism comes into play). This suggests that an attribute can be represented as a function from individuals to propositions, that is, as the simplest kind of propositional function. Suppose the attribute has uniqueness built into it, so that it can be attributable to no more than one individual per world. Let us, for the moment, think of propositions as sets of possible worlds rather than as characteristic functions of such sets. Then the propositional function F, which represents an attribute with uniqueness built in, never assigns compatible propositions (i.e., those containing a common possible world) to distinct individuals. Thus, for a given world w there is at most one individual x such that $w \epsilon F(x)$. The propositional function F is now barely distinguishable from an individual concept. By a slight variation on Russell's scheme for representing attributes—adding uniqueness—we see how to represent an individual concept as a function from individuals to propositions. (A final caveat: the propositional function must, of course, have all possible individuals in its domain. This can be achieved either by assuming a fixed domain of individuals common to all possible worlds or by taking the values of the individual variables to be all possible individuals and adding a predicate to distinguish the actual. It also seems possible to treat the case of overlapping domains without quantifying over possible individuals, but new ideas are required and the result is less simple.)

An individual concept c will be represented by that propositional function F which assigns to a possible individual x exactly that set of worlds w such that $c(w) = x$. *This idea generalizes.* Concepts of entities of type α_1 can be represented by functions from (possible) entities of type α to propositions.

The role of Haecceitism in this reduction is apparent. In models that are isomorphic (in the Anti-Haecceitist's sense), corresponding individual concepts will not necessarily be reduced to corresponding propositional functions. The particular propositional function to which an individual concept is reduced depends on the transworld identities.

The result of applying this reduction to entities of sucessively higher intensional types is that we can ultimately represent all of the entities of Churcho within the sub-ontology whose types are just ι, o, o_1, and $(\alpha\beta)$ for any types α and β which are already in-

cluded. In this development, the only basic intensional entities that remain are the propositions.

VII

Two final points are worthy of note. First, the concept that is the Fregean sense of a definite description like 'the site of Troy' is reduced to the propositional function F such that if x is any individual, $F(x)$ is the proposition: *that x is a unique site of Troy*. (Of course, for the Anti-Haecceitist, there is no such proposition; the place in the that-clause occupied by the variable must be filled by a sense-bearing name, and the place in the singular proposition occupied by x must be filled by an individual concept.) This is in accord with Russell's views about definite descriptions and provides us with his solution to the earlier-mentioned problem regarding such intensional verbs as 'seeks', which do not take a sentential complement. *Seeks* becomes a relation between an individual and an attribute (I do not here distinguish between attributes and the corresponding propositional functions), for example, between Schliemann and the attribute of *being a unique site of Troy*. Frege's analysis seems superior only so long as we focus on "Schliemann sought the site of Troy" in which 'seek' takes a singular term as direct object, and ignore "Schliemann sought a wealthy benefactor." Russell would regard the second form, in which 'seeks' takes an indefinite noun phrase as complement, as showing the *deep* structure for 'seeks', and analyzes the first on that model.

Finally, I note that the basic technique for reducing the hierarchy of intensions to propositional functions is quite general. It is not limited to the kind of intensions generated by the possible-worlds methodology, wherein two expressions have the same intension if they are logically equivalent. If we find Haecceitism acceptable—and most writers seem to do so—this raises the prospect that in attempting to construct new intensional logics (for example, logics with a more fine-grained principle of individuation for intensions) we can concentrate our efforts on the much simpler Russellian form.[6]

DAVID KAPLAN

University of California, Los Angeles

[6] The work reported above is an outgrowth of and was inspired by the seminar "Recent Developments in the Logic of Sense and Denotation" given in fall 1973 by Professor Alonzo Church. It has benefited from the comments and suggestions of several persons. The deficiencies of the present report are due, in part, to the severe time constraints under which it was prepared. Its merits are due, in part, to the "instant criticism" of Robert Adams, Tyler Burge, Montgomery Furth, Donald Kalish, and Joseph Lambert. Both the thought and the writing were supported by the National Science Foundation.