

## THE NUMBER OF THINGS

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1. I wish to attack a certain idea—roughly speaking, the idea that it is nonsensical to speak of the number of objects. I begin with two long quotations that express this view, or variants on it. The first is from Wittgenstein's *Tractatus*<sup>1</sup>, the second from Putnam's *The Many Faces of Realism*<sup>2</sup>.

4.1271 Every variable is the sign of a formal concept.

For every variable represents a constant form that all its values possess, and this can be regarded as a formal property of those values.

4.1272 Thus the variable name 'x' is the proper sign for the pseudo-concept *object*.

Wherever the word 'object' ('thing', etc.) is correctly used, it is expressed in conceptual notation by a variable name.

For example, in the proposition, 'There are two objects which ...', it is expressed by '( $\exists x, y$ )...'

Wherever it is used in a different way, that is as a proper concept-word, nonsensical pseudo-propositions are the result.

So one cannot say, for example, 'There are objects', as one might say, 'There are books'. And it is just as impossible to say, 'There are 100 objects', or, 'There are  $\aleph_0$  objects'.

And it is nonsensical to speak of the *total number of objects*.<sup>3</sup>

Conceptual relativity sounds like 'relativism', but has none of the 'there is no truth to be found ... "true" is just a name for what a bunch of people can agree on' implications of 'relativism'. A simple example will illustrate what I mean. Consider 'a world with three individuals' (Carnap often used examples like this when we were doing inductive logic together in the early nineteen-fifties), **x1**, **x2**, **x3**. How many *objects* are there in this world?

Well, I *said* "consider a world with just three individuals", didn't I? So mustn't there be three objects? Can there be non-abstract entities which are not 'individuals'?

One possible answer is 'no'. We can identify 'individual', 'object', 'particular', etc., and find no absurdity in a world with just three objects which are independent, unrelated 'logical atoms'. But there are perfectly good logical doctrines which lead to different results.

Suppose, for example, that like some Polish logicians, I believe that for every two particulars there is an object which is their sum. (This is the basic assumption of ‘mereology’, the calculus of parts and wholes invented by Lezniewski.) If I ignore, for the moment, the so-called ‘null object’, then I will find that the world of ‘three individuals’ (as Carnap might have had it, at least when he was doing inductive logic) actually contains *seven* objects:

<b>World 1</b>	<b>World 2</b>
<b>x1, x2, x3</b>	<b>x1, x2, x3, x1+x2, x1+x3, x2+x3, x1+x2+x3</b>
<b>(A world à la Carnap)</b>	<b>(‘Same’ world à la Polish logician)</b>

Some Polish logicians would also say that there is a ‘null object’ which they count as a part of every object. If we accepted this suggestion, and added this individual (call it **O**), then we would say that Carnap’s world contains *eight* objects.

Now, the classic metaphysical realist way of dealing with such problems is well-known. It is to say that there is a single world (think of this as a piece of dough) which we can slice into pieces in different ways. But this ‘cookie cutter’ metaphor founders on the question, ‘What are the parts of this dough?’ If the answer is that **O, x1, x2, x3, x1+x2, x1+x3, x2+x3, x1+x2+x3** are all the different ‘pieces’, then we have not a *neutral* description, but rather a *partisan* description—just the description of the Warsaw logician! And it is no accident that metaphysical realism cannot really recognize the phenomenon of conceptual relativity—for that phenomenon turns on the fact that *the logical primitives themselves, and in particular the notions of object and existence, have a multitude of different uses rather than one absolute ‘meaning’.*

In these passages, Wittgenstein and Putnam defend similar conclusions—perhaps the same conclusion, although whether their conclusions are strictly the same is a nice question. Their arguments, however, are certainly different.

I will first make some remarks that pertain to their common conclusion—if their conclusions are the same—or apply equally to their two similar conclusions.

(1) Suppose that one accepts the following two theses about sets. First, that the words ‘set’ and ‘member’ express, respectively, a unique attribute and a unique relation that everyone familiar with the language of set theory grasps, and that every sentence in the language of set theory, in consequence, expresses a determinate proposition—in the case of the simpler sentences, a proposition that everyone familiar with the language of set theory grasps. Secondly, that all the sentences in the language of set theory that are theorems of Zermelo-Fraenkel set theory express true propositions. One is then, as we may say, a “set-theoretical realist,” and, if numbers and sets are objects, the set-theoretical realist will regard it as nonsensical to speak of the total number of objects—or, if not nonsensical at any rate demonstrably incoherent, since it is a theorem of Zermelo-Fraenkel set theory that for every number there is a set containing a greater number of objects than that number. (We could state this theorem informally in these words: There are too many objects for them to be numbered.)

Whether these theses and their consequences are true or not, the sense in which they entail that there is no such thing as the number of objects is not the sense either Wittgenstein or Putnam intended to give to these words. As to Wittgenstein, he would certainly not have accepted anything remotely resembling set-theoretical realism. He would, moreover, have argued that ‘There are more than 100 objects’ was nonsensical for the same reason that ‘There are 100 objects’ was nonsensical—and the set-theoretical realist will certainly want to say that the former sentence expresses a truth. As to Putnam, his point obviously does not depend on an appeal to facts, if there are facts, about sets or any other abstract objects. His argument contains no premise that contradicts the strictest nominalism.<sup>4</sup> I will therefore assume in the sequel that if there is no such thing as the total number of objects, this is true for some reason other than that there are too many of them to be numbered.

(2) Suppose (as I have argued is at least possible<sup>5</sup>) there is such a thing as vague identity. That is, suppose that the following is possible: for some  $x$  and for some  $y$ , it is indeterminate whether  $x = y$ . If there were such a thing as vague identity, there would be no such thing as the total number of objects—at any rate, no number would be such that it was definitely the number of objects. Here is a simple case:

$\exists x \exists y$  (it is indeterminate whether  $x = y$  and  $\forall z$  (it is indeterminate whether  $z = x$  or it is indeterminate whether  $z = y$  or it is determinately true that  $z = x$  or it is determinately true that  $z = y$ )).

If we are asked what the number of objects is in any of the rather sparsely populated universes in which this sentence is true, we can say the following and no more than the following: it is indeterminate whether the number of objects is 1; it is indeterminate whether the number of objects is 2; for every other number (including 0), it is determinately true that that number is not the number of objects.

This point, however, is unrelated to the arguments of Wittgenstein and Putnam. In the sequel, I will assume that when we raise the question whether there is such a thing as the total number of objects, we are asking this question about a universe in which identity is not vague. (Wittgenstein, as we shall see, denies that there is any such relation as identity. This is consistent with my assumption, for identity is vague only if there is such a relation as identity.)

(3) Suppose that “identity is always relative to a sortal term.” Suppose, that is, that there is no such thing as identity *simpliciter*, but only a two-or-more-membered class of symmetrical and transitive relations each of which is expressible by a sentence of the form ‘ $x$  is the same N as  $y$ ’, where ‘N’ represents the place of a sortal term (or at least that this is true of each member of the class that is expressible in English). And suppose that there are sortals

M and N such that “For some  $x$  and  $y$ ,  $x$  is the same M as  $y$  and  $x$  is not the same N as  $y$ ” is true.<sup>6</sup> If this is the case, then there is no such thing as counting or numbering *simpliciter*; there is only counting or numbering by Ns. Suppose for example, that there are two such relations, “is the same being as” and “is the same person as”. Suppose that there exist an  $x$ , a  $y$ , and a  $z$  such that none of  $x$ ,  $y$ , and  $z$  is the same person as the others, each of  $x$ ,  $y$ , and  $z$  is the same person as itself [i. e., each is a person], and each of  $x$ ,  $y$ , and  $z$  is the same being as the others; suppose, too, that everything has this feature: it is the same being as one of  $x$ ,  $y$ , and  $z$  and it is the same person as one of  $x$ ,  $y$ , and  $z$ .<sup>7</sup> In a universe whose population is given by this description, there is no answer to the question, ‘How many objects are there?’ The best one could do in reply to the question would be to say something like, “Well, how do you want me to count them? Counting objects by beings, there is exactly one. Counting objects by persons, there are exactly three.” (There was no need to appeal to Trinitarian theology to make this point. The point could have been made in terms of ‘is the same gold statue as’ and ‘is the same piece of gold as’—provided one was willing to say that  $x$  might be the same piece of gold as  $y$  but not the same gold statue as  $y$ .)

This point, again, is unrelated to the arguments of Wittgenstein and Putnam. In the sequel, I will assume that when we raise the question whether there is such a thing as the total number of objects, we are assuming that there is no case in which  $x$  is the same M as  $y$  but not the same N; I will assume that statements of the form ‘ $x$  is the same M as  $y$ ’ are to be understood as the corresponding statements of the form ‘ $x$  is M and  $x = y$ ’ where ‘=’ represents the classical identity-relation: the relation whose logical properties are given by these two conditions: it is reflexive; it forces indiscernibility—that is, if  $x$  is identical with  $y$ , then whatever is true of  $x$  is true of  $y$ . Again, my assumption is consistent with Wittgenstein’s thesis that there is no such relation as identity—that is to say, his assumption that the identity-sign is meaningless. If the only way to understand the expression ‘ $x$  is the same M as  $y$ ’ is as a stylistic variant on ‘ $x$  is M and  $x = y$ ’, and if the identity-sign is meaningless, all that follows is that ‘ $x$  is the same M as  $y$ ’ is meaningless.

So: we will assume that if there is no such thing as the number of objects, this is not because there are too many objects for them to be counted, and we will assume that identity is neither vague nor “relative to a sortal term.” Given these assumptions, is there some reason to suppose that it is nonsensical to speak of the total number of objects? Let us first examine Wittgenstein’s argument for this conclusion.

2. I divide Wittgenstein’s argument into two parts, the first ending with ‘it is expressed by “ $(\exists x, y) \dots$ ”, and the second comprising the remainder of the passage I have quoted. Although I am not entirely sure I understand everything Wittgenstein says in the first part of the argument, it seems plausible to me to suppose that he is saying something very much like this:

‘Object’ is simply an unrestricted count-noun, a count-noun of maximal generality. An object is anything that can be the value of a variable, that is, anything that we can talk about using pronouns, that is *anything*. These two points—that ‘object’ is an unrestricted count-noun and that an “object” is anything that can be the value of a variable can be combined in the following observation: the word ‘object’ is so used that any substitution-instances of the following pair of formulae are equivalent:

$$\forall x (x \text{ is an object} \rightarrow Fx) \quad \forall x Fx;$$

and of the following pair:

$$\exists x (x \text{ is an object} \& Fx) \quad \exists x Fx.$$

Thus, the word ‘object’ is a mere stylistic convenience: anything we can say using this word we can say without using it; it can be dispensed with in favor of variables—or pronouns.

If this is (more or less) what Wittgenstein is saying in the first part of the argument, then I (more or less) agree with him. But what is said in the second part of the argument does not seem to follow from what is said in the first—nor does it seem to be true.<sup>8</sup> *Why* can one not say that there are objects?<sup>9</sup> Why not say it this way: ‘ $\exists x x = x$ ’? And why can one not say ‘There are at least two objects’ like this:

$$\exists x \exists y \sim x = y,$$

or say ‘There are exactly two objects’ like this:

$$\exists x \exists y (\sim x = y. \& \forall z (z = x \vee z = y))?$$

(If we had sufficient patience and sufficient paper, we could say ‘There are at least 100 objects’ and ‘There are exactly 100 objects’ in similar fashion.) The identity-sign would seem to express what Wittgenstein calls a formal concept, and, it would seem, it can be combined with quantifiers and variables (which express the formal concept “object”) to say how many objects there are.

Wittgenstein’s reply to this objection lies in his account of the identity-sign in 5.53–5.532, which in effect says that ‘=’ means nothing and must be banished from a “correct conceptual notation.” (Russell in his *Introduction to the Tractatus* (p. xvii) sees very clearly the connection between what Wittgenstein says about the total number of objects and what he says about the symbol ‘=’.)

Let us examine what Wittgenstein says in 5.53–5.532. Proposition 5.53, on which the rest of the passage is a commentary, is

Identity of object I express by identity of sign, and not by using a sign for identity.  
Difference of objects I express by difference of signs.

Now consider someone who has mastered the treatment of identity in any standard logic text of the present day, and who is reading the *Tractatus* for the first time. This reader will probably want to protest that it is simply wrong to suppose that “difference of objects” *can* be expressed by “difference of signs”—or that statements apparently asserting the difference of objects can be translated into statements in which what had apparently been expressed by the negation of an identity-statement was to be expressed by using different signs. If we examine 5.532, which reads in part

... I do not write ‘ $\exists x \exists y (Fxy \ \& \ x = y)$ ’, but ‘ $\exists x Fxx$ ’; and not ‘ $\exists x \exists y (Fxy \ \& \ \sim x = y)$ ’, but ‘ $\exists x \exists y Fxy$ ’.

(I have replaced Wittgenstein’s *Principia* notation with current logical notation), we shall see what troubles our imaginary reader: although the first two formulae are logically equivalent, the second two are not. But this reaction on the part of the imaginary reader is premature, as the remainder of 5.532 shows. Wittgenstein is not proposing to replace a formula with a formula that is not logically equivalent to it; he is rather proposing an alternative logical notation. In the proposed “Tractarian notation,” ‘ $\exists x \exists y Fxy$ ’ does not mean what it means in standard notation, but rather what ‘ $\exists x \exists y (Fxy \ \& \ \sim x = y)$ ’ means in standard notation. What is expressed by ‘ $\exists x \exists y Fxy$ ’ in standard notation is expressed in Tractarian notation by

$$\exists x Fxx. \vee \exists x \exists y Fxy.^{10}$$

Wittgenstein says no more than this about how Tractarian notation is to work, but it is not hard to extend the line of thought hinted at in 5.532.

What is expressed in standard notation by ‘ $\exists x \exists y \exists z Fxyz$ ’ will be expressed in Tractarian notation by

$$\exists x Fxxx. \vee \exists x \exists y Fxyy. \vee \exists x \exists y Fxyx. \vee \exists x \exists y Fxxy. \vee \exists x \exists y \exists z Fxyz.$$

And what is expressed in Tractarian notation by ‘ $\exists x \exists y \exists z Fxyz$ ’ is expressed in standard notation by

$$\exists x \exists y \exists z (Fxyz \ \& \ \sim x = y \ \& \ \sim x = z \ \& \ \sim y = z).$$

The difference between standard and Tractarian notation has been usefully described by Hintikka in these words: the variables of standard notation are “inclusive”; the variables of Tractarian notation are “exclusive.”<sup>11</sup>

Using exclusive variables, the sentence ‘There are at least two chairs’ may be rendered as

$$\exists x \exists y (x \text{ is a chair} \ \& \ y \text{ is a chair}),$$

and ‘There are at most two chairs’ as

$$\sim \exists x \exists y \exists z (x \text{ is a chair} \ \& \ y \text{ is a chair} \ \& \ z \text{ is a chair}).$$

‘There are exactly two chairs’ is then represented by the conjunction of these two formulae. The situation is similar with the universal quantifier. The Tractarian formula ‘ $\forall x Fxx \ \& \ \forall x \forall y Fxy$ ’ replaces the standard ‘ $\forall x \forall y Fxy$ ’, and the Tractarian ‘ $\forall x \forall y Fxy$ ’ is equivalent to the ordinary ‘ $\forall x \forall y (\sim x = y \rightarrow Fxy)$ ’.

How far can this technique be extended? Consider the standard language of first-order logic, with the identity sign, but with no terms but variables. Can all closed sentences in this language be translated into Tractarian notation? Well, certainly not all. Not

$$(1) \ \forall x x = x,$$

or

$$(2) \ \forall x \exists y (Fx \ \& \ x = y).$$

But it would seem that all standard formulae that do not contain an “unpredicated” occurrence of a variable can be translated into Tractarian notation. (An occurrence of a variable is unpredicated if it occurs beside an occurrence of ‘=’ and no occurrence of the same variable is one of the string of occurrences of variables following a predicate-letter within the scope of the quantifier that binds it. The second and third occurrences of ‘x’ in (1) are thus unpredicated; the second occurrence of ‘y’ in (2) is unpredicated. But the third occurrence of ‘x’ in (2) is “predicated” because another occurrence of ‘x’ follows the predicate-letter ‘F’, and that occurrence falls within the scope of the quantifier that binds the third occurrence of ‘x’.) The reader is referred to the article by Hintikka cited in note 11 for a systematic statement of rules for translating formulae in standard notation into Tractarian notation. Hintikka also gives systematic rules for translating formulae in Tractarian notation into standard notation. If a standard formula is translated into Tractarian notation by the first set of rules, and the resulting Tractarian formula translated into standard notation by the second set of rules, the result will not in general be the original standard formula; but it will always be provably equivalent to the original standard formula.

What is the philosophical meaning of this result? Let us call those formulae in standard notation that cannot be translated into Tractarian notation—like (1) and (2) above—“untranslatable.” The standard formulae that I offered above as ways of saying ‘There are objects’ [ $\exists x x = x$ ], and ‘There are exactly two objects’ [ $\exists x \exists y (\sim x = y \ \& \ \forall z (z = x \vee z = y))$ ] are untranslatable; each contains—in fact, quantifier phrases aside, contains only—unpredicated occurrences of variables.<sup>12</sup> But why should this trouble those who believe that

untranslatable formulae are meaningful? The fact that something cannot be translated into a particular notation does not necessarily mean that it is nonsense; it may mean only that that notation is deficient in expressive power. Wittgenstein does not argue that untranslatable formulae are meaningless simply because they cannot be translated into Tractarian notation. The general form of his argument is rather this: the identity-sign is strictly meaningless; therefore the statements in which it occurs can be meaningful only if it is a mere notational convenience, eliminable in principle. But why does Wittgenstein think the identity-sign is strictly meaningless? First, because the *Principia* definition of identity (an identity-of-indiscernibles definition:  $x = y$  if and only if everything that is true of  $x$  is true of  $y$  and everything that is true of  $y$  is true of  $x$ ) is inadequate, owing to the fact that it is at least *possible* for there to be two distinct things that share all their attributes (like Kant's two raindrops). Secondly,

5.5303 Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all.

As to the first point, it could be debated at great length whether there could be two things that were indiscernible. I will not, however, enter into this debate. Let us suppose that Wittgenstein is right on this point. It can hardly be true that a sign that has no explicit definition must for that reason be meaningless, or all signs would be meaningless. (Just as—as Wittgenstein said later in his career—explanations come to an end somewhere, so explicit definitions come to an end somewhere.) Why can we not simply define '=' as shorthand for 'is identical with'—perhaps coupled with an informal discussion of "numerical" versus "descriptive" identity? Or one might define identity as a universally reflexive relation that forces indiscernibility. Admittedly, for all anyone can say, there might be two relations that satisfied this definition; but the following is provable: if '='<sub>1</sub> expresses one of these relations, and '='<sub>2</sub> the other, then

$$\forall x \forall y (x =_1 y \leftrightarrow x =_2 y).$$

(The fact that there might be two such relations leads *me* to regard this definition as unacceptable. The definition does not, or at least may not, tell us what relation identity *is*. I am willing to accept the definition of '=' as 'is identical with'—a phrase we, as speakers of English, understand—and to say that "a universally reflexive relation that forces indiscernibility" embodies a correct theory of the logically valid inferences that can be made using 'is identical with'.) As to the argument of 5.5303, I think it suffices to point out that we do other things with 'is identical with' than say things like 'Tully is identical with Cicero and Tully and Cicero are two things' or 'Tully is identical with Tully'. I will not rest my case on the supposed informativeness of 'Tully is identical with Cicero', for Wittgenstein—at least this seems to me to be what he should say—will reply that the only informative proposition that this sentence could

express is the proposition that something is called both ‘Tully’ and ‘Cicero’, and I do not wish to enter into the questions this reply would raise. I will point out instead that when one makes assertions using complex closed sentences that contain expressions like ‘ $z = x$ ’ and ‘ $\sim y = z$ ’, what one thereby does fits neither of the following descriptions: ‘saying of two things that they are identical’; ‘saying of one thing that it is identical with itself’. Whether or not sentences like ‘ $\exists x \exists y (\sim x = y. \& \forall z (z = x \vee z = y))$ ’ are meaningful, it is evident that the person who uses them can neither be said to be asserting of two things that they are identical nor be said to be asserting of something that it is identical with itself.

A parenthetical remark: I am convinced that Tractarian notation can be explained only in terms of standard notation—that the concept of an exclusive variable can be grasped only by someone who has a prior and independent grasp of the concept of numerical non-identity. But any way I can think of to argue for this conclusion would be called (and probably rightly) circular, so I shall not press this point.

It seems doubtful that Wittgenstein’s contention that ‘=’ is strictly meaningless is right. But let concede, for the sake of the argument, that ‘=’ is meaningless. What follows? It does not follow that one cannot say that there are objects or cannot say how many objects there are—although it does of course follow that one cannot employ ‘=’ in any essential way in making these assertions. Some other device will be needed. No such device can be found within the language of pure logic, for the only formulae of pure logic that can (on anyone’s account) be used to make assertions contain the identity-sign: this two-place predicate is the only predicate that belongs to the language of logic, and one cannot make assertions without using predicates. To assert the existence of and to count “objects,” we need an open sentence that everything satisfies. All such sentences that the language of pure logic affords—in fact, all sentences of any kind that the language of pure logic affords—contain ‘=’. But the language of “impure” logic, which comprises sentences in which quantifiers bind variables that occur in expressions containing words belonging to some natural language, affords an unlimited supply of open sentences that are satisfied by everything. And there is no rule that restricts us to the language of pure logic. Here is a proposal for representing the assertion ‘There are objects’ in which the formal concept “object” is expressed by means of variables:

$\exists x (x \text{ is a chair} \vee \sim x \text{ is a chair}).$

What objections can be brought against this proposal? Well, there is an aesthetic objection: the choice of the predicate ‘is a chair’ is entirely arbitrary, and thus the definition can hardly be put forward as a paradigm of elegance. But aesthetical deficiencies really do not bulk much larger in judging philosophical theses than they do in judging plans for rescuing a child trapped in a well: the only really important question in either case is: Will it work?

It is possible that someone will object to this proposal on the ground that ‘is a chair’ is a vague predicate, and that the vagueness of ‘ $x$  is a chair’ is inherited by ‘ $x$  is a chair  $\vee \sim x$  is a chair’ (it may be that something that is a borderline-case of “chair” does not determinately satisfy ‘ $x$  is a chair  $\vee \sim x$  is a chair’; determinately to satisfy this predicate, it might be argued, is determinately to satisfy one or the other of its disjuncts). And vagueness causes problems for counting. We need not examine the merits of this objection, however, for there are predicates that are determinately satisfied by everything, whether or not ‘ $x$  is a chair  $\vee \sim x$  is a chair’ is one of them. Here is one: ‘ $x$  is evenly divisible by 3  $\vee \sim x$  is evenly divisible by 3’. (I take it that ‘ $\sim$  Frederick the Great is evenly divisible by 3’ expresses a truth. Frederick certainly does not belong to the extension of ‘ $x$  is evenly divisible by 3’.) Since this “out” is available, I will assume in what follows that ‘ $x$  is a chair  $\vee \sim x$  is a chair’ is determinately satisfied by everything. Having this predicate at our disposal, we can say, for example, ‘There are exactly two objects’ using only language that meets all Wittgenstein’s requirements. Using Tractarian notation (that is, exclusive variables) we render ‘There are at least two objects’ as follows:

$\exists x \exists y (x \text{ is a chair } \vee \sim x \text{ is a chair. } \& .y \text{ is a chair } \vee \sim y \text{ is a chair}).$

And we render ‘There are at most two objects’ as

$\sim \exists x \exists y \exists z (x \text{ is a chair } \vee \sim x \text{ is a chair. } \& .y \text{ is a chair } \vee \sim y \text{ is a chair. } \& .z \text{ is a chair } \vee \sim z \text{ is a chair}).$

‘There are exactly two objects’ is, of course rendered as the conjunction of these two sentences.

“To say, ‘Frederick the Great is a chair  $\vee \sim$  Frederick the Great is a chair’ is to say nothing at all.” (Cf., “... to say of *one* thing that is identical with itself is to say nothing at all.” See 5.513.) Well, it’s certainly to say nothing at all controversial. (“So tell your Papa where the Yak can be got,/And if he is awfully rich,/He will buy you the creature—/Or else he will not./I cannot be positive which.”—Hilaire Belloc.) But it is not to say nothing at all in the sense in which to utter any of the following sentences would be to say nothing at all:

It is now five o’clock on the sun

Boggle the main franistan

*Das Nichts nichtet*

Turn right, and the entrance is diagonally opposite, by the next street.<sup>13</sup>

Instances of the law of the excluded middle frequently appear as (obviously meaningful) lines in mathematical proofs—e.g., ‘The number  $N$  is either prime

or it is not prime; In the latter case, it is not the greatest prime; In the former, it is also (as we have seen) composite, which is a contradiction; Therefore, there is no greatest prime.’ I conclude that there is no merit in this objection.

We can, therefore, if we have world enough and time, write a sentence that expresses any of the propositions in the following infinite sequence:

There are no objects

There is exactly one object

There are exactly two objects

There are exactly three objects

.  
.  
.

Each of these sentences is meaningful and has a truth-value (given that identity is neither vague nor relative to a sortal term). At most one of them is true. If one of them *is* true, it gives the number of objects. If none of them is true, then there are infinitely many objects. In the latter case, if we allow ourselves the apparatus of transfinite numbers (if we allow ourselves, say, the language of Zermelo-Fraenkel set theory) we can say, for any number  $x$ , that the number of objects is  $x$ ; we say: There is a set of objects such that  $x$  is the number of that set’s members, and no number greater than  $x$  is the number of the members of any set of objects. And any such statement will be meaningful and will have a truth-value, since every set has a cardinal number and every cardinal number is either the cardinal number of a given set or it isn’t. (If we do not allow ourselves the apparatus of transfinite numbers, and if all the sentences in our sequence are false, then admittedly we can’t say what the number of objects is. But we can say that they are *numberless*: that for any number  $x$ , there are at least  $x+1$  objects. And this will be a statement that is true without qualification. But, if Wittgenstein is right, this statement is as meaningless as ‘There are exactly 100 objects.’) Now if “object” is, as we have been supposing, a formal concept—so that sets and numbers are by definition objects—all sentences of the form ‘ $x$  is the number of objects’ will be false, owing to the fact that no distinction can be made between sets of objects and sets *tout court* (which do not form a set and hence have no number). But suppose we restrict the question of the number of objects to “individual” or “non-abstract” objects, as Putnam does. Then it would seem that the number of “objects” must be 0 or 1 or 2 or ... or infinite. If it is infinite (and if, as it seems reasonable to suppose, there are not too many *individuals* for them to be numbered), and if we allow ourselves the apparatus of transfinite numbers, then *some* number must be the number of objects—at least assuming that we can refer without paradox to the set of all things that satisfy ‘ $x$  is an individual or non-abstract object’.<sup>14</sup> After all,

every set has a cardinality. But if Putnam’s argument is right, this conclusion must be wrong. Let us examine his argument.

3. What can Putnam mean when he says that “the logical primitives themselves, and in particular the notions of object and existence, have a multitude of different uses rather than one absolute ‘meaning’.”? This sentence presupposes the truth of several theses. One of them is that the notion of an object is a “logical primitive.” What does this mean? Not, obviously, that some symbol used in formal logic is a logical primitive in the sense that, say, ‘ $\sim$ ’ is, and, moreover, bears the same relation to the English word ‘object’ that ‘ $\sim$ ’ bears to ‘it is not the case that’. Perhaps the best way to understand the idea that the notion of an object is a logical primitive—I should think the only way to understand this idea—is to equate it with Wittgenstein’s idea that “object” is a formal concept: that anything one can say using the word ‘object’ one can say without using it; it can be dispensed with in favor of variables—which is to say, in favor of third-person-singular pronouns. But then what can it mean to say that ‘object’ has “a multitude of different uses rather than one absolute ‘meaning’.”? Have variables “a multitude of different uses rather than one absolute ‘meaning’.”? Have third-person-singular pronouns?

We might understand the thesis that variables have a multitude of different uses rather than one absolute meaning as the familiar thesis that variables are essentially “sorted,” that each “logical category” requires its own style of variable. But different styles of variable are a mere notational convenience. If we like, we can use, say, bold-face variables for, say, sets, and ordinary italic variables for other objects, but this is only a labor-saving device. It allows us to write

$$\exists \mathbf{x} \exists y \sim \exists z (y \in \mathbf{x} \ \& \ \sim z \in \mathbf{x})$$

in place of

$$\exists x \exists y \sim \exists z (x \text{ is a set} \ \& \ y \text{ is a set} \ \& \ \sim z \text{ is a set.} \ \& \ y \in x \ \& \ \sim z \in x).$$

And “unsorted” variables are what we must start with, for a variable is in essence a third-person-singular pronoun, and there is only one-third-person-singular pronoun, and it has only one meaning. We do not have one third-person-singular pronoun for talk about objects in one logical category and another for talk about objects in another, and we do not use ‘it’ with one sense when we are talking about artifacts and with another when we are talking about numbers or laws or amounts of money or trade routes. If these things were not so, the following sentences would be nonsense:

Everything has this property: if it’s not a proper class, then it’s a member of some set

No matter what “logical category” a thing belongs to, it can’t have contradictory properties

If something belongs to the extension of a predicate, it can do so only as the result of a linguistic convention.

And these sentences are quite plainly *not* nonsense. It is therefore hard to see what Putnam can mean by saying that ‘object’ has a multitude of different uses. (He is certainly not saying that the word ‘object’ is used in more and less restricted ways—that in many contexts we use ‘object’ as an abbreviation for ‘... object’ or ‘object that is F’. He is not saying that the “world à la Carnap” and the “world à la Polish logician” really have the same inhabitants, and that Carnap and the Polish logician are merely using two sets of linguistic conventions, conventions according to which Carnap restricts the range of his variables to three of the inhabitants of the seven- or eight-membered world and the Polish logician does not restrict the range of his variables.)

If this argument is correct, a parallel argument would seem to apply to what Putnam says about ‘existence’. If existence is a “logical primitive” (that is, if the words ‘exist’ and ‘existence’ can be dispensed with in favor of ‘ $\exists$ ’) it cannot have “a multitude of different uses.” If it did, number-words like ‘three’ and ‘six’ and ‘forty-three’ would have a multitude of different uses. It is evident, however, that Carnap and the Polish logician do not mean different things by ‘three’ when Carnap says ‘There are exactly three objects’ and the logician says, ‘There are more than three objects’. It is not possible to suppose that Carnap and the Polish logician mean different things by the formula

$$\exists x \exists y \exists z (\sim x = y. \& \sim x = z. \& \sim y = z. \& \forall w (w = x \vee w = y \vee w = z)).$$

If Carnap and the Polish logician differ about whether this formula is true in some world, this cannot be because they mean different things by this formula—or mean different things by ‘three’. They cannot mean different things by this formula because there is only one thing for this formula *to* mean. (Well, one could give it “different meanings” in a sense: one could place various restrictions on the range of its variables. But, again, this cannot be what Putnam is talking about when he says that “existence has a multitude of different uses rather than one absolute ‘meaning’.”)

It is therefore not at all clear what Putnam’s conclusion is. But perhaps we can understand his conclusion if we examine his argument. (It is usually a good strategy to examine an argument when you do not understand its conclusion.) The argument has to do with counting things. But what sort of thing, or what sorts of things, are being counted in Putnam’s argument? I will begin my attempt to find an answer to this question by listing the count-nouns that Putnam uses in connection with counting:

—individual

—object

—(non-abstract) entity [used only once, in connection with the suggestion that there might be objects that were not individuals]

—particular

—logical atom [used only once, and in scare-quotes]

—part [used only once, in connection with the “cookie cutter” metaphor: ‘What are the parts of the dough?’].

The most important of these count-nouns would seem to be ‘individual’, ‘object’, and ‘particular’. Apparently, the relation between these three terms is more or less as follows:

We are discussing only non-abstract things—“particulars.” Among the particulars, perhaps co-extensive with them, are “individuals.” But it may be that there are “objects” that are particulars but not individuals. ‘Object’ is the most general of the three count-nouns: everything is an object; particulars are non-abstract objects. But since we are not discussing abstract objects, since we have excluded them from our universe of discourse, ‘object’ and ‘particular’ in effect coincide. We can, therefore, let the word ‘particular’ drop out of the discussion, and simply ask whether there are objects that are not individuals.

I am afraid I have no sense of what the distinction between an individual and a (non-abstract) object that is not an individual is supposed to be. When we try to see what Putnam is getting at in attempting to distinguish individuals and objects that are not individuals, we find only one clue: it seems that if  $x$  and  $y$  are individuals, then  $x + y$  is an object that is not an individual. But I do not find that this clue leads me anywhere. Let me try to explain why. I will begin by discussing the symbol ‘+’. This symbol is a “term-maker”: it takes two terms and makes a term. What does this term-maker mean? It is not normally (in developments of “mereology”) taken as a primitive. It is normally defined in terms of some other mereological predicate—say, ‘overlaps’ or ‘is a part of’ (‘part’ being used in the “inclusive” sense in which everything is a part of itself). Let us take ‘is a part of’ as primitive. Using this predicate, we first define ‘ $x$  overlaps  $y$ ’ or ‘ $x$  and  $y$  have a common part’ in the obvious way. We may then write

$$F x + y =_{df}$$

$\exists!z (Fz \text{ and } x \text{ is a part of } z \text{ and } y \text{ is a part of } z \text{ and } \forall w (\text{if } w \text{ is a part of } z, \text{ then } w \text{ overlaps } x \text{ or } w \text{ overlaps } y)).$

This definition can be generalized. We can define a more general operator, an operator on sets, ‘the sum of’ or ‘the sum of the members of’. The operators

‘+’ and ‘the sum of’ are related in the obvious way: the sum of  $\{\mathbf{x1}, \mathbf{x2}\} = \mathbf{x1} + \mathbf{x2}$ , and the sum of  $\{\mathbf{x1}, \mathbf{x2}, \mathbf{x3}\} = \mathbf{x1} + \mathbf{x2} + \mathbf{x3}$ . (And the sum of  $\{\mathbf{x1}, \text{the sum of } \{\mathbf{x2}, \mathbf{x3}\}\} = \mathbf{x1} + (\mathbf{x2} + \mathbf{x3})$ —which, as Putnam’s notation suggests is just  $\mathbf{x1} + \mathbf{x2} + \mathbf{x3}$ ; in mereology, as in arithmetic, when using ‘+’ we can move or remove brackets as we will.)<sup>15</sup>

The operators ‘+’ and ‘the sum of’ are used by “the Polish logician” in connection with a theory called ‘mereology’. Putnam calls mereology a “logical doctrine” and “the calculus of parts and wholes invented by Lezniewski.” But that is like calling Zermelo-Fraenkel set theory a logical doctrine and “the calculus of sets and members invented by Zermelo and Fraenkel.” Mereology is in no sense a part of logic and it is certainly misleading to call something that makes such intransigent existential claims a “calculus.” As set theory is a theory about members and sets (that is, a theory about the membership relation), so mereology is a theory about parts and wholes (that is, a theory about the parthood relation). And mereology is a *particular* theory about parts and wholes. Different theories of parts and wholes are possible, theories that differ from one another far more than competing versions of set theory differ from one another. Competing versions of set theory differ about rather esoteric matters. Any two versions of set theory agree that if  $x$  and  $y$  are two individuals, then the set  $\{x, y\}$  exists. Competing theories of parts and wholes disagree about such fundamental matters as whether, if  $x$  and  $y$  exist,  $x + y$  exists. Consider, for example, the theory of parts and wholes I have called Nihilism,<sup>16</sup> whose sole axiom is ‘Nothing has any proper parts’<sup>17</sup> or ‘Parthood is coextensive with identity’. A second theory of parts and wholes that is inconsistent with mereology can be obtained by stipulating that any set has *at least* one mereological sum (and thereby leaving open the possibility that some sets have two or more mereological sums); by stipulating, that is, that for any such set, there will be *at least* one object that has all that set’s members as parts and all of whose parts overlap some member of that set (see note 15). We could call this theory “Pluralism.” Pluralism will be congenial to those philosophers who maintain that a gold statue can be distinct from the lump of gold from which it is made. For if the statue and the lump are distinct objects, then they are distinct mereological sums of the same gold atoms: the statue has certain gold atoms as parts and every part of the statue overlaps at least one of those atoms; the lump has those same gold atoms as parts and every part of the lump overlaps at least one of them.

When we consider Nihilism and Pluralism—the former denies the numerically distinct objects  $x$  and  $y$  a mereological sum and the latter allows them to have *two* mereological sums—, we can see why I have used the words “competing theories of parts and wholes” rather than the words “competing versions of mereology.” Mereology with and without the null individual can sensibly be called competing versions of one theory. Nihilism, Pluralism, and any version of mereology are competing theories, full stop. To emphasize the fact that mereology is a particular theory about parts and wholes, one of many competing theories, I will in the sequel spell ‘mereology’ with a capital ‘M’. Mereology has two axioms: that parthood is transitive and that any non-empty set has a

(unique) mereological sum.<sup>18</sup> (Perhaps it should be mentioned that some advocates both of Mereology and Pluralism may want to regard the quantifier “any set” as restricted in some way, since they may think that there are objects whose nature unfits them for being parts—they may want to say, for example, that only individuals, or only concrete objects, or only material objects, can be parts. To accommodate such possible scruples, we could understand “any set” as “any set all of whose members are ontologically suited to being parts of something.” This reading would not affect any of the points at issue in this discussion.)

Now that we know what is meant by Mereology, let us examine “the world à la Carnap” and “the world à la Polish logician.” The former is supposed to contain three individuals. Putnam’s language (‘independent’, ‘unrelated’) pretty clearly suggests that these three individuals are not supposed to overlap mereologically—they are not supposed to have any parts in common. (And not only is this suggested by his language, it must be his intent. If, say,  $x_1$  were  $x_2 + x_3$ , then Putnam’s Carnap and the Polish logician would agree about the number of objects: they would both say that there were three; the Polish logician will count seven or eight objects only if  $x_1$  and  $x_2$  and  $x_3$  do not overlap one another.) Putnam’s language (“logical atoms”) also suggests that  $x_1$  and  $x_2$  and  $x_3$  have no proper parts, that they are mereological simples. (And not only is this suggested by his language, it must be his intent. If any of them did have proper parts, these proper parts would themselves be individuals—or so I would suppose, but I’m feeling my way about in the dark here—, and, assuming “no overlap,” there would be *more* than three individuals in the world à la Carnap.) Let us suppose, therefore that the world à la Carnap contains exactly three simples. These would be “Carnap”’s “three individuals.” It is a theorem of Mereology that if a world contains exactly three simples, it also contains exactly four composite objects (non-simples, objects with proper parts) and contains nothing else. (From now on I will ignore the null individual, owing simply to fact that I can make no sense whatever of the idea of a null individual.) Are composite objects, objects with proper parts, not individuals? Are the mereological sums of individuals not themselves individuals? Why on earth not? If Putnam’s Carnap says that a world that contains exactly three simples contains exactly three objects or exactly three individuals full stop, then he must reject Mereology—he must contend that Mereology is a false theory. And the “Polish logician” must hold that the description ‘a world that contains three simples and nothing else’ is an impossible description. (Of course, the friends of Mereology will be perfectly happy with the description ‘a world that contains three individuals and nothing else’; this description would be satisfied by a world that contained exactly two simples,  $x_1$  and  $x_2$ ; this world would have exactly one other inhabitant,  $x_1 + x_2$ . And if  $x_1$  and  $x_2$  are individuals, no doubt  $x_1 + x_2$  will also be an individual. How not?) It makes perfect sense to ask, Who (if either) is right, “Carnap” or “the Polish logician”? It makes perfect sense to ask, “Could there be a world that contained nothing but three simples?” If Mereology is a true theory about the part-whole relation, the answer is No. If Mereology is a false theory about the part-whole relation, the answer may well be Yes.

Since Mereology is a theory, we are free to reject it—in the absence of compelling reasons for accepting it or at least for regarding it as plausible. As it happens, I reject it. (I regard it, in fact, as wholly implausible.) At least: I reject it if ‘is a part of’ in the statement of the theory means what ‘is a part of’ means in English. (And I do not know what else it could mean.) Mereology makes assertions about what there is, and I do not accept these assertions. Take, for example, my dog Sonia and my cat Moriarty. If Mereology is a true theory, then there is such a thing as the sum of Sonia and Moriarty. What properties does this object have? The theory itself tells us only that it has Sonia and Moriarty as parts and that each of its parts overlaps either Sonia or Moriarty—and that it has such other properties as may be logically derivable from these. But I know some things about Sonia and Moriarty, and I know some things about parthood (e.g., that if a point in space falls inside a part of a thing all of whose parts are extended in space, then it falls inside that thing; that if  $x = y + z$  and  $y$  and  $z$  do not overlap, then the mass of  $x$  is equal to the sum of the masses of  $y$  and  $z$ ). It follows from Mereology and these things I know that there exists a scattered object<sup>19</sup> that weighs about twenty-five pounds and has two maximally connected parts<sup>20</sup> each of which is now asleep, is about forty feet from the other, and is covered with fur. I do not believe there is any such thing, since I do not believe anything has these properties. Just as those who believe that I have no immaterial soul believe this because they think that nothing has the set of properties a thing would have to have to be my soul, so I think that nothing is the sum of Sonia and Moriarty because I think that nothing has the set of properties a thing would have to have to be that sum. And why should one think there was any such thing? After all, that there is a theory that says there is something with certain properties is, taken by itself, a rather unimpressive reason for believing that there is something that has those properties. I can, if I like, put forward a theory (“soul theory”) that says that every mental property is instantiated only by something that also has the property immateriality, but if you think that nothing has both the property *is thinking about Vienna* and the property immateriality, you are unlikely to believe my theory. And I don’t believe Mereology—any more than I believe Nihilism. Although I don’t deny that some sets of material objects have sums, I don’t think a very high proportion of them do. For most sets of, say, atoms, I don’t think that there is anything that has the set of properties that the sum of that set of atoms would have to have. Putnam’s Polish logician and I disagree not only about simple, imaginary worlds, but about the real world. We mean the same thing by ‘mereological sum’, since we mean the same thing by ‘is a part of’, which is no technical term but a term of ordinary English. (Or very close to it. Perhaps the English phrase ‘is a part of’ means what ‘is a proper part of’ means in the language of Mereology.) The “Polish logician” and I simply disagree about what mereological sums there are; like the atheist and the theist, the dualist and the materialist, and the nominalist and the platonist, we disagree about what there is. The “Polish logician” and I use the definite description ‘Sonia + Moriarty’ in the

same sense; he thinks something has the right properties to be the denotation of this phrase and I don't.

I cannot, therefore, grant that "Carnap"'s and the "Polish logician"'s descriptions are equally good or equivalent descriptions of the population of a world—not, at least, if Carnap's description is 'a world that contains three mereological simples and nothing else'. I cannot grant that they *could* be equally good or equivalent descriptions of the population of a world, for they are straightforwardly incompatible, as incompatible as 'a world that contains immaterial souls' and 'a world that contains only material things'. Putnam's argument, therefore, is, as I have understood it, incoherent. It is, of course, possible that I have *not* understood it. There are two ways in which this might have happened. One of them is that there is something there to be understood and I have failed to understand it.<sup>21</sup>

## Notes

1. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus: The German text of Ludwig Wittgenstein's Logisch-Philosophische Abhandlung, with a new Translation by D. F. Pears & B. F. McGuinness, and with the Introduction by Bertrand Russell, F.R.S.* (London: Routledge & Kegan Paul, second impression, 1963). In citations, I will use Wittgenstein's "proposition numbers," rather than page numbers.
2. Hilary Putnam, *The Many Faces of Realism: the Paul Carus Lectures* (La Salle, Illinois: Open Court, 1987). The quoted passage occurs on pp. 17–19.
3. Proposition 4.1272 does not end at this point.  
See also 4.128 and 5.453. Proposition 4.128 reads

Logical forms are *without* number.

Hence, there are no privileged numbers in logic, and hence there is no possibility of philosophical monism or dualism, etc.

By 'monism' and 'dualism', Wittgenstein of course means the thesis that there is one object (as Spinoza held) and the thesis that there are two objects (a thesis that perhaps no one has held)—not the thesis that there is one *kind* of object (as materialists and idealists hold) and the thesis that there are two *kinds* of object (as Descartes held).

4. It might be that there were more nominalistically acceptable objects than could be numbered. This could not happen if all nominalistically acceptable objects were spatio-temporal objects and were within the same space-time. But suppose there were more space-times than could be numbered (a supposition consistent with David Lewis's modal ontology). Then there could be more objects extended in space and time—there could, in fact, be more tables—than could be numbered. In the sequel I shall simply assume that if it is impossible to number the nominalistically acceptable objects, this is not because there are too many of them. (And we assume that there are no non-existent objects of any sort: I should think that if there *were* non-existent tables and elephants and neutron stars, it would be entirely plausible to suppose that there were too many of them to be numbered.)
5. *Material Beings* (Ithaca, N.Y.: Cornell University Press, 1990), §18.

6. (I use bold-face double-quotes for “Quine corners” or “quasi-quotation-marks.”) This assumption requires us to assume that the following two theses do not hold:

“For all  $x$  and for all  $y$ , if  $x$  is the same  $M$  as  $y$  and  $F...x...$ , then  $F...y...$ ”

“For all  $x$  and for all  $y$ , if  $x$  is the same  $N$  as  $y$  and  $F...x...$ , then  $F...y...$ ”,

where ‘ $F...x...$ ’ is a sentence in which ‘ $y$ ’ does not occur and ‘ $F...y...$ ’ is the result of replacing some or all of the free occurrences of ‘ $x$ ’ in ‘ $F...x...$ ’ with ‘ $y$ ’. Roughly speaking: if the analogue of the principle of the indiscernibility of identicals held for all “relative identities,” one could not have a case in which relative identities did not “coincide.”

7. I did not have to use English phrases like ‘the others’ and ‘one of’ to say this; all that is required to say it is the apparatus of quantifier logic, the usual sentential connectives, and the predicates ‘ $1$  is the same being as  $2$ ’ and ‘ $1$  is the same person as  $2$ ’.
8. Carnap seems to assert that the second part of the argument does not follow from the first—see *The Logical Syntax of Language* (London: Routledge & Kegan Paul, 1937), p. 295—but I am not sure I understand Carnap’s point.
9. Wittgenstein says that one cannot say ‘“There are objects”, as one might say, ‘There are books’.’ I have no idea what the words ‘as one might say’ [‘*wie man etwa sagt*’] could mean, so I will ignore them.
10. In standard notation, ‘ $\exists x \exists y Fxy$ ’ is, of course, equivalent to

$$\exists x Fxx. \vee \exists x \exists y (Fxy \ \& \ \sim x = y).$$

11. Jaakko Hintikka, “Identity, Variables, and Impredicative Definitions,” *The Journal of Symbolic Logic*, 21 (1956), pp. 225–245.
- 12.

5.534 And now we see that in a correct conceptual notation pseudo-propositions like ‘ $a = a$ ’, ‘ $a = b \ \& \ b = c. \rightarrow a = c$ ’, ‘ $\forall x x = x$ ’, ‘ $\exists x x = a$ ’, etc. cannot even be written down.

(As before, I have replaced Wittgenstein’s *Principia* notation with current logical notation.)

13. Suppose that the second of these sentences is uttered by someone as a parody of nautical terminology. The fourth is from Stephen Potter’s *Lifemanship*. It is recommended for inclusion in a letter giving instructions to a visiting team on how to find the home team’s playing field. It has, according to Potter, the following annoying virtue: the fact that it is meaningless will not become evident to the visiting team till they have exhausted themselves in the attempt to find the home team’s playing field.
14. And assuming that ‘individual’ is meaningful and does not admit of borderline cases. There may well be problems about the meaning of ‘individual’—but if there are, Putnam’s argument does not appeal to them. (Is ‘individual’ perhaps ambiguous? If

so, pick one of its possible senses and concentrate on that one. If Putnam is right, there can be no meaning to the question ‘How many objects are there in a world containing three individuals?’ when the word ‘individual’ is used in that sense.) And if the word ‘individual’ is meaningful, it’s hard to see how there could be borderline cases of individuals; at any rate, Putnam’s argument does not have as a premise that ‘individual’ is vague.

15. The more general definition is:

$$F(\text{the sum of } S) =_{\text{df}}$$

$$\exists!z (Fz \text{ and every member of } S \text{ is a part of } z \text{ and every part of } z \text{ overlaps some member of } S).$$

It should be noted that we do not *have* to suppose that the word ‘sum’ can occur only within a definite description. If we wished, we could read the predicate ‘every member of *S* is a part of *z* and every part of *z* overlaps some member of *S*’ as ‘*z* is a sum of *S*’. That would allow us to say that the members of a set had *more than one* mereological sum.

16. *Material Beings*, §8.
17. A proper part of an object is a part of that object other than the whole object—for in the formal theory of parts and wholes, it is convenient to regard every object as being by definition a part of itself.
18. As Putnam has said, some versions of mereology recognize a “null individual,” the sum of the empty set. In those versions, the second axiom is that every set has a sum.
19. That is an object that is not “all in one piece”: a spatial object having at least two parts that are such that every path through space that joins those two parts passes outside that object.
20. A connected object is an object that is not a scattered object: an object that is “all in one piece.” A maximally connected object is a connected object that is not a proper part of a connected object. A maximally connected part of an object *x* is a connected part of *x* that is not a proper part of any connected part of *x*. If there are cats and (undetached) cats’ tails, then a cat’s tail is a connected part of the cat, but not a *maximally* connected part, since there are connected parts of the cat—the cat itself if no other—of which it is a part. If a dog and a cat (spatially separated in the ordinary way; not surgically joined or anything special like that) have a mereological sum, then the cat is a maximally connected part of that sum, since there is no *connected* part of the sum that has the cat as a proper part. (Couldn’t we simply define a maximally connected part of *x* as a part of *x* that is a maximally connected object? No: if a cat’s head and tail have a sum, the tail is a maximally connected part of the sum, but is not a maximally connected object.)
21. In “Truth and Convention: On Davidson’s Refutation of Conceptual Relativism,” *Dialectica* 41 (1987), pp. 69–77, Putnam imagines some criticisms of the lessons he draws from the confrontation between “Carnap” and “the Polish Logician,” and these criticisms bear at least some resemblance to my criticisms. The criticisms are put into the mouth of a “Professor Antipode,” a figure of fun. (Professor Antipode, like most figures of fun, is not very intelligent.) I believe that the resemblance is

superficial. However this may be, I do not understand Putnam's reply to Professor Antipode. The reader must judge: either I am obtuse in the extreme, or (inclusive) the words of Putnam's reply to Professor Antipode, like the words of his original argument, cease even to seem to mean anything when they are subjected to careful analysis.

In *Representation and Reality* (Cambridge, Mass.: MIT Press, 1988), p. 110 ff, Putnam makes a point that could be applied to the confrontation between Carnap and the Polish Logician in this way: the whole dispute is really about which things to apply the word 'object' to, and that dispute is to be settled by establishing a convention. (He goes on to attempt to "deconstruct" the fact/convention distinction. I am sorry to be boring about this, but I am afraid I shall have to say once more that I do not understand the words this attempt comprises. I am aware that "I don't understand" is used by many philosophers as a substitute for argument, but I really *don't* understand what he says.) I have in effect replied to this argument (leaving aside the attempted deconstruction of the fact/convention distinction, which I am not in a position to say anything about) in *Material Beings*, pp. 6–12. The essence of my argument was this: if a thing doesn't exist, it isn't *there* for you to establish a convention to the effect that it shall be called an 'object' (or anything else); if it does exist, the term 'object' applies to it, since the term applies to everything.