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'Whether' report

KNOWING WHETHER AND TELLING WHETHER

Mr. Body lies foully murdered, and the suspects are Green, Mustard, Peacock, Plum, Scarlet, and White. We may take it as settled that one of them did it, and only one. The question is whether Green did it, or Mustard did it, or Peacock, or Plum, or Scarlet, or White. Holmes is on the scene.

If Green did it, then Holmes knows whether Green did it or . . . or White did it if and only if he knows that Green did it. Likewise if Mustard did it, then Holmes knows whether . . . if and only if he knows that Mustard did it. Likewise for the other cases. In short, Holmes knows whether . . . if and only if he knows the true one of the alternatives presented by the 'whether'-clause, whichever one that is.

Similarly for telling. In at least one principal sense, Holmes tells Watson whether Green did it, or Mustard did it, or Peacock, or Plum, or Scarlet, or White, if and only if Holmes tells Watson the true one of the alternatives presented by the 'whether'-clause. That is: if and only if either Green did it and Holmes tells Watson that

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An ancestor of this paper was written in 1974 as a contribution to a Workshop on Semantics and Syntax of Non-Extensional Constructions, held at the University of Massachusetts, Amherst, and sponsored by the Mathematical Social Sciences Board. I am grateful to the participants for their comments, to the University for hospitality, and to the Board for support.

Green did it, or . . . or White did it and Holmes tells Watson that White did it.

This is a *veridical* sense of telling whether, in which telling falsely whether does not count as telling whether at all, but only as purporting to tell whether. This veridical sense may or may not be the only sense of *'tell whether'*; it seems at least the most natural sense.

'Whether'-clauses may be abbreviated. Holmes knows or tells whether Green or . . . or White did it if and only if he knows or tells whether Green did it or . . . or White did it. He knows or tells whether Green did it or not if and only if he knows or tells whether Green did it or Green did not do it. He knows or tells whether Green did it if and only if he knows or tells whether Green did it or not. And of course we may also abbreviate by putting plain commas in place of all but the last of the '*or*'s.

IGNORANCE, SECRECY, WONDERING, AND ASKING WHETHER

Some other constructions with 'whether' may be analyzed in terms of knowing or telling whether. Lestrade's ignorance as to whether Green did it or . . . or White did it consists in his not knowing whether Green did it or . . . or White did it. He remains ignorant because Holmes keeps it secret from him whether Green did it or . . . or White did it, and Holmes does so exactly by knowing whether while declining to tell him whether. Consequently Lestrade wonders whether Green did it or . . . or White did it; that is (1) he does not know whether Green did it or . . . or White did it; (2) he desires an end to this ignorance; and (3) this desire is part of his conscious thought.

When Holmes told Watson whether Green did it or . . . or White did it, that was because Watson had asked him. Watson had requested that Holmes tell him whether. Or perhaps Watson, knowing Holmes' distaste for the straight answer, rather had requested that Holmes see to it somehow that Watson come to know whether.

In terms of Åqvist's imperative-epistemic analysis of questions, given in [6], [7], [8], and [9], Watson's question to Holmes could be formalized as

Let it be that: you see to it that: I soon come to know: whether (Green did it, . . . , White did it)

where this ought to come out equivalent to

Let it be that: you see to it that: (I soon come to know that Green did it $\vee \ldots \vee$ *I soon come to know that White did it).*

Three comments. First, we need not make it explicit that the one to become known is to be the true one, for only truth can possibly become known. Second, the operator of agency *'you see to it that'* and the adverb *'soon'* are not present, or at any rate not explicit, in Åqvist's own formulation; but, as noted in [2], they seem to be desirable – though complicating – additions. Third, we need to do something to stop the implication to

Let it be that: you see to it that: (Green did it $\vee \ldots \vee$ *White did it),*

or else to reinterpret this consequence so that it does not amount to a request on Watson's part that Holmes bring about a past murder. The problem may have various solutions; Åqvist presents one solution (credited to Hintikka) in [9].

A close relative of Åqvist's analysis is the imperative-assertoric analysis discussed *inter alia* in [2]. It seems preferable in many cases, though perhaps (given that Watson takes account of Holmes' penchant for making things known otherwise than by telling them) the original imperative-epistemic analysis better suits our present case. Be that as it may, an imperative-assertoric analysis of Watson's question could run as follows.

Let it be that: you tell me: whether (Green did it, . . . , White did it)

where this ought to come out equivalent to

Let it be that: ((Green did it & you tell me that Green did it) $\vee \ldots \vee$ *(White did it & you tell me that White did it)).*

Two of our previous comments apply, *mutatis mutandis*. First, the request imputed to Watson on this analysis is again somewhat redun-

dant. He could simply have requested that Holmes tell him some one of the alternatives, without requesting that Holmes pick the true one. Indeed Holmes *could* have told him one of the others; but Holmes *would* not have done so, being an honourable man dealing with a close friend. Watson was in a position to rely on Holmes' truthfulness without especially requesting it – indeed, had he not been, neither could he have relied on Holmes to be truthful on request. The same is true generally: those questioned are supposed to tell the truth without any special request. (On this point I am indebted to conversation with John Searle.) But it is harmless to impute to Watson a needlessly strong request; and I do so in order to connect the question '*Tell me whether . . . ?*' with the veridical sense of *'tell whether'*. Second, we need to do something to stop the implication to

Let it be that: (Green did it $\vee \ldots \vee$ *White did it),*

or else we need to reinterpret this conclusion so that it is not a malevolent optative.

In short: Watson's question was a request that Holmes tell him, or at any rate that Holmes somehow make known to him, the true one of the alternatives presented by the 'whether'-clause.

STRATEGIES FOR SEMANTIC ANALYSIS

Suppose that we wished to provide a formal semantic analysis for a fragment of English that includes the constructions just considered: constructions of knowing and telling whether, and other constructions analyzable in terms of these. How might we treat 'whether'-clauses? At least five alternative strategies come to mind.

A. *We might eliminate 'whether'-clauses altogether.* Rather than taking *'whether'* plus its appended list of alternatives as a constituent, we might take *'whether'* as part of the verb. Thus *'know whether'*, *'tell whether'*, and *'ask whether'*, for instance, would be verbs that attach to lists of sentences and in some cases also to indirect objects to make intransitive verb phrases. These verbs would be semantically indivisible primitives: their semantic values would not be derived from

the semantic values of *'know'*, *'tell'*, and *'ask'*, nor would the word *'whether'* have any semantic value of its own. Hence this strategy passes up opportunities for unification both in its treatment of different constructions with *'know' et al.* and in its treatment of different constructions with *'whether'*.

B. *We might assign 'whether'-clauses to a special category all their own,* or to a special category reserved for wh-clauses generally. This would at least permit a unified treatment of 'whether'; but the treatment of *'know' et al.* remains disunified, since we must distinguish the 'know' that attaches to a 'whether'-clause from the categorically different *'know'* of *'Holmes knows that Peacock is innocent'*. And, other things being equal, we should not multiply categories.

C. *We might treat 'whether'-clauses as terms denoting their sets of alternatives.* Now we have a unified treatment of *'whether'* and we need no special category. But we still have a disunified treatment of *'know'*: sometimes the knower is related to a single propositional content (or content of some other appropriate sort), sometimes rather to a set of alternative contents. Likewise for *'tell'*. Also, we have a new problem, avoided by strategies A and B: a term denoting a set of alternative contents ought to make sense in various positions where a 'whether'-clause cannot in fact occur:

* *Whether Green did it or Plum did it has two members.*

D. *We might treat 'whether'-clauses as terms denoting the true one of the alternatives they present.* Now we get full unification, at least for the constructions so far considered. If Green did it, then we may take *'whether Green did it or White did it'* and *'that Green did it'* as two terms denoting the same thing, and capable of being attached to the same verb *'know'* or *'tell'*. But again we need arbitrary-seeming restrictions to stop the occurrence of 'whether'-clauses in positions where terms denoting the true one of the alternatives ought to make sense:

* *Whether Mustard or Scarlet did it is true,*
* *Whether Mustard or Scarlet did it is some sort of abstract entity.*

Of course, the same difficulty arises to some extent with 'that'-clauses regarded as denoting terms:

* *That Plum did it is some sort of abstract entity.*

E. *We might treat 'whether'-clauses as sentences expressing the same content as the true one of the presented alternatives.* If we do not wish to treat 'that'-clauses as denoting terms, perhaps because of such difficulties as the one just noted, we could treat *'know'* and *'tell'* as verbs that attach to sentences. Then the word *'that'* would not be assigned any semantic value, though syntax would still have to take account of it. This strategy imitates those formal languages that contain epistemic or assertoric modal operators. If we do this, and we still seek unification, 'whether'-clauses also must be treated as sentences; and the content of the 'whether'-clauses must be the same as that of the true alternative. We shall see how this is possible.

The problem of arbitrarily prohibited occurrences is still with us, though our previous examples of it are disposed of when we no longer treat 'whether'-clauses as terms. For instance, why can't we say

* *Lestrade believes whether Mustard or Scarlet did it*

to mean that Lestrade believes the true one of the alternatives thereby presented?

Strategies D and E have the advantage of unifying the treatment of *'know'* and *'tell'*; but C, and perhaps also B, may do better at unifying the treatment of *'whether'* itself. For there is a second family of constructions with *'whether'* besides the ones we have considered so far; and in these, what matters is not especially the true one of the alternatives but the whole range of them. For these constructions, the step to denoting (or expressing) the true one of the alternatives is a step in an unhelpful direction. This second family consists of constructions expressing dependence or independence: *'whether . . . or . . . depends on . . . ', '. . . depends on whether . . . or . . . ', 'no matter whether . . . or . . . , still . . . ', 'it doesn't matter whether . . . or . . . '*, and so on.

So the advantages are not all on one side; and I would not wish to resurrect the 'whether' report that I once wrote (and then suppressed) which definitely advocated strategy E. Nevertheless, it remains of interest that strategy E is even possible; and it does have its attractions. Let us examine it further.

For the remainder of this paper, I shall suppose that *'whether'* is a sentential operator of variably many places: it attaches to two or more sentences to make a sentence. The sentences it attaches to are punctuated by *'or'*s, perhaps supplemented or replaced at all but the final occurrence by commas.

I shall also suppose that the content expressed by a sentence (in context) is the same sort of thing that can be known or that can be told; and that it is the same thing as the *truth set* of the sentence: the set of maximally specific possibilities that make the sentence true (under the interpretation bestowed on it by its actual context). It is common to take these possibilities as abstract representations of ways the whole world might be; I am inclined to disagree on two counts, taking them rather as concrete possible individuals. But for the sake of neutrality, let me call them simply *points.* A sentence, then, is true at some points, false at others; and the content it expresses is the set of points at which it is true.

We want a 'whether'-clause to express the same content as does the true one of the sentences embedded in it, provided that exactly one of those sentences is true. But how can we provide for that? If A is the true one of A and B, and we assign the same content to *'whether A or B'* as we do to A itself, then that assignment would have been wrong if B had been the true one! The answer is that we must not assign content to 'whether'-clauses once and for all; rather, we must make the content vary, depending on contingent matters of fact.

That means that we must resort to *double indexing,* a technical device invented by Hans Kamp and Frank Vlach in the late 1960's and since used in many ways by many authors. (See, for instance, [1], [3], [4], [5], and [10].) We shall not say that a sentence is true at a point *simpliciter;* rather, a sentence is true at a point relative to a second point, which may or may not be the same as the first. For most sentences, the second point is idle: *normal* sentences satisfy the condition:

$\models_{i,j} A$ if and only if $\models_{i,k} A$, for any j and k.

It will therefore be useful to reintroduce truth at a single point by way of abbreviation, noting that this will be the only truth relation that matters for normal sentences:

$\vDash_i A$ is defined as $\vDash_{i,i} A$.

Content of a sentence relative to a point is then defined as the set of first points that make the sentence true relative to the fixed second point:

$[\![A]\!]_j = {}^{df}\{i: \vDash_{i,j} A\}$.

A normal sentence has the same content relative to all points; an abnormal sentence does not. Sentences made with *'whether'* are, in general, abnormal. They have variable content, as is required by strategy E. Our rule for *'whether'* is as follows.

$\vDash_{i,j}$ *whether* A_1 *or* . . . *or* A_n
if and only if either $\vDash_i A_1$ and $\vDash_j A_1$
or . . . or $\vDash_i A_n$ and $\vDash_j A_n$.

If the A's themselves are normal, and if exactly one of them is true at point j, then we have the desired result: the content relative to j of the 'whether'-clause is the same as the content relative to j (or absolutely) of the true one of the A's:

If $\vDash_j A_1$, $\nvDash_j A_2, \ldots, \nvDash_j A_n$, then $[\![$*whether* A_1
or . . . *or* $A_n]\!]_j = [\![A_1]\!]_j$;

and so on for the other cases.

Now if we have the natural rules for *'know'* and *'tell'*, so that for instance

$\vDash_i$ *Holmes knows (that)* A if and only if Holmes at point i stands in the knowing-relation to the content $[\![A]\!]_i$

then we will get the right results for knowing whether, telling whether, and whatever is definable in terms of them. Putting our 'whether'-clause in as the sentence A (and relying on our syntax to delete, or not to insert, the word *'that'*) we find as expected that, for instance

> If $\vDash_i A_1$, $\nvDash_i A_2$, . . . , $\nvDash_i A_n$, then $\vDash_i$ *Holmes knows whether* A_1 *or . . . or* A_n if and only if Holmes at point i stands in the knowing-relation to the content $[\![A_1]\!]_i$;

and so on for the other cases.

Given our double indexing, it is really not quite enough to give conditions of truth *simpliciter* for sentences such as '*Holmes knows whether* A_1 *or . . . or* A_n', '*Holmes tells Watson whether* A_1 *or . . . or* A_n', and their relatives. We must also say when such a sentence is true at a point i relative to a second point j. The simplest stipulation, I think, is that our sentences of knowing and telling are to be normal sentences:

> $\vDash_{i,j}$ *Holmes knows (that)* A if and only if
> $\vDash_i$ *Holmes knows (that)* A

with the right-hand side evaluated according to the condition of truth *simpliciter* already given. This applies, in particular, to '*Holmes knows whether* A_1 *or . . . or* A_n'. Accordingly, our sentences of knowing whether and telling whether can without trouble be embedded into further 'whether'-clauses, as in

> *Holmes tells Watson whether or not Lestrade knows whether Plum did it or Green did it.*

What happens if, at a certain point j, *none* of the alternatives presented in the 'whether'-clause is true?

> If $\nvDash_j A_1$, . . . , $\nvDash_j A_n$, then $\nvDash_{i,j}$ *whether* A_1 *or . . . or* A_n

so in that case the content relative to j of the 'whether'-clause is the empty set – the content that cannot possibly be true. This impossible content cannot be known; so if none of $A_1, \ldots, A_n$ is true at j, then it cannot be true at j that Holmes knows whether A_1 or . . . or A_n. And that seems exactly right. Also it seems that in this case it cannot be true at j that Holmes tells Watson whether A_1 or . . . or A_n (at any rate, not in our veridical sense of telling whether). This too is a consequence of our rules, provided that the empty set – the impossible content – is something that cannot be told. I think that indeed the empty set cannot be told. (Contradicting oneself should not count as telling one thing with impossible content, but rather as telling two or more things with conflicting contents. Or so I think – but this is highly controversial.) If so, our rule performs as it should in this case too.

What happens if, at a certain point j, *more* than one of the presented alternatives are true? Then I think a presupposition required for the proper use of the 'whether'-clause has failed, with the result that no clear-cut data on truth conditions are available and so we may as well handle the case in the most technically convenient way. The interested reader is invited to discover how I have implicitly handled the case.

What if some of the sentences embedded in the 'whether'-clause are already abnormal, expressing different content relative to different points? Again, the interested reader may discover my implicit answer. But I think the question idle, for there is no reason to believe that the difficulty need ever arise. It would arise if 'whether'-clauses were immediately embedded in other 'whether'-clauses – but that should not be allowed to happen in any correct fragment of English. Perhaps it might arise if our double indexing for 'whether'-clauses interacted with double indexing introduced for some other purpose; but we could always stop such interaction by moving to triple, quadruple, . . . indexing.

'WHETHER'-CLAUSES AS DISJUNCTIONS

Strategy E offers an answer to one question that its rivals do not address. Why the *'or'* in *'whether Green did it or Scarlet did it'*? We need a punctuation mark to separate the listed sentences that give the alter-

natives, but why do we use *'or'* for the job? Why not always commas? Or some special mark just for this purpose? Why does a 'whether'-clause look like a disjunction?

(Of course, we must be careful. We must distinguish *'Holmes knows whether Green did it or Scarlet did it'* from *'Holmes knows whether (Green did it or Scarlet did it)'*. The latter, with its straightforward disjunction, does not present *'Green did it'* and *'Scarlet did it'* as separate alternatives. Rather, it abbreviates *'Holmes knows whether (Green did it or Scarlet did it) or not-(Green did it or Scarlet did it)'*, and the alternatives presented are the disjunction and its negation. If Holmes knew that it was Green or Scarlet but didn't know which, then the latter sentence would be true but the former false.)

Strategy E offers this answer: a 'whether'-clause *is* a disjunction. Let us introduce an operator *'wheth'* which is similar to *'whether'* except that it attaches to a single sentence rather than a list of two or more. It makes abnormal sentences; its semantic rule is a simplification of the rule for *'whether'*, as follows.

$\vDash_{i,j}$ *wheth A* if and only if $\vDash_i A$ and $\vDash_j A$.

Or, in terms of content, if A is normal:

If $\vDash_j A$, then $[\![\textit{wheth } A]\!]_j = [\![A]\!]_j$;

If $\nvDash_j A$, then $[\![\textit{wheth } A]\!]_j$ is the empty set.

Then a 'whether'-clause is equivalent to a truth-functional disjunction of 'wheth'-clauses; *'whether'* is *'wheth'* plus *'or'*. For instance:

$\vDash_{i,j}$ *whether A or B* if and only if $\vDash_{i,j}$ *wheth A or*
$\vDash_{i,j}$ *wheth B*
if and only if $\vDash_{i,j}$ (*wheth A* $\vee$ *wheth B*).

Of course, this is a bit artificial; in that respect the original form of strategy E, without the reduction of *'whether'* to *'wheth'*, is more attractive.

RECOVERING THE ALTERNATIVE SET

I noted that the step to strategy E (or even D) was a step in the wrong direction so far as constructions of dependence and independence are concerned. But it is not a fatally wrong step, for it is reversible should the need arise. Without treating the second family of constructions in any detail, I shall simply note that given the function that acts as the semantic value of a 'whether'-clause in strategy E – namely, the function that gives the truth value at any pair of points, or alternatively the function that gives the content relative to any one point – we can recover the alternative set needed in treating the constructions in the second family. Let W be a 'whether'-clause; then the set of alternatives presented by W is $\{[\![W]\!]_j$: j a point $\}$, that is,

$$\{X{:}\exists j\ X = \{i{:}\ \vdash_{i,j} W\}\}.$$

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