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ORDERING SEMANTICS AND PREMISE SEMANTICS FOR COUNTERFACTUALS*

1. COUNTERFACTUALS AND FACTUAL BACKGROUND

Consider the counterfactual conditional 'If I were to look in my pocket for a penny, I would find one'. Is it true? That depends on the factual background against which it is evaluated. Perhaps I have a penny in my pocket. Its presence is then part of the factual background. So among the possible worlds where I look for a penny, those where there is no penny may be ignored as gratuitously unlike the actual world. (So may those where there is only a hidden penny; in fact my pocket is uncluttered and affords no hiding place. So may those where I'm unable to find a penny that's there and unhidden.) Factual background carries over into the hypothetical situation, so long as there is nothing to keep it out. So in this case the counterfactual is true. But perhaps I have no penny. In that case, the absence of a penny is part of the factual background that carries over into the hypothetical situation, so the counterfactual is false.

Any formal analysis giving truth conditions for counterfactuals must somehow take account of the impact of factual background. Two very natural devices to serve that purpose are orderings of worlds and sets of premises. Ordering semantics for counterfactuals is presented, in various versions, in Stalnaker [8], Lewis [5], and Pollock [7]. (In this paper, I shall not discuss Pollock's other writings on counterfactuals.) Premise semantics is presented in Kratzer [3] and [4]. (The formally parallel theory of Veltman [11] is not meant as truth-conditional semantics, and hence falls outside the scope of this discussion.) I shall show that premise semantics is equivalent to the most general version – roughly, Pollock's version – of ordering semantics.

I should like to postpone consideration of the complications and disputes that arise because the possible worlds are infinite in number. Let us therefore pretend, until further notice, that there are only finitely many worlds. That pretence will permit simple and intuitive formulations of the theories under consideration.

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2. ORDERING SEMANTICS IN THE FINITE CASE

We may think of factual background as ordering the possible worlds. Given the facts that obtain at a world i, and given the attitudes and understandings and features of context that make some of these facts count for more than others, we can say that some worlds fit the facts of world i better than others do. Some worlds differ less from i, are closer to i, than others. Perhaps we do not always get a clear decision between two worlds that depart from i in quite different directions; the ordering may be only partial. And perhaps some worlds differ from i so much that we should ignore them and leave them out of the ordering altogether. But sometimes, at least, we can sensibly say that world j differs less from i than world k does.

In particular, i itself differs not at all from i, and clearly no world differs from i less than that. It is reasonable to assume, further, that any other world differs more from i than i itself does.

Given a counterfactual to be evaluated as true or false at a world, such an ordering serves to divide the worlds where the antecedent holds into two classes. There are those that differ minimally from the given world; and there are those that differ more-than-minimally, gratuitously. Then we may ignore the latter, and call the counterfactual true iff the consequent holds throughout the worlds in the former class.

The ordering that gives the factual background depends on the facts about the world, known or unknown; how it depends on them is determined – or underdetermined – by our linguistic practice and by context. We may separate the contribution of practice and context from the contribution of the world, evaluating counterfactuals as true or false at a world, and according to a 'frame' determined somehow by practice and context.

We define an *ordering frame* as a function that assigns to any world i a strict partial ordering $\angle_i$ of a set S_i of worlds, satisfying the condition:

(Centering) i belongs to S_i; and for any j in S_i, $i \angle_i j$ unless $i = j$.

(A *strict partial ordering* of a set is a transitive, asymmetric, binary relation having that set as its field.) We call j a *closest A-world to i* (according to an ordering frame) iff (i) j is an A-world, that is, a world where proposition A holds, (ii) j belongs to S_i, and (iii) there is no A-world k such that $k \angle_i j$. We can lay down the following truth condition for a counterfactual from A to C (that is, one with an antecedent and consequent that express the propositions A and C, respectively): the counterfactual is true at world i, according to an ordering frame, iff

(OF) C holds at every closest A-world to i.

Truth condition OF is common to all versions of ordering semantics, so long as we stick to the finite case; what I have said so far is neutral between the theories of Stalnaker, Lewis, and Pollock.

The three theories differ in various ways. Some of the differences concern informal matters: is it correct or misleading to describe the orderings that govern the truth of counterfactuals as comparing the 'overall similarity' of worlds? Just how is the appropriate ordering determined by features of the compared worlds, and by practice and context? How much residual indeterminacy is there? These questions fall mostly outside the scope of this paper. Other differences concern extra formal requirements that might be imposed on ordering frames. So far, following Pollock, we have allowed merely partial orderings, in which worlds may be tied or incomparable. Lewis permits ties but prohibits incomparabilities; Stalnaker prohibits ties and incomparabilities both. We shall consider these disagreements in Section 5. Further differences concern the postponed difficulties of the infinite case, and we shall consider these in Section 6.

The restricted field S_i of the ordering $\angle_i$ might well be a needless complication. We could get rid of it by imposing an extra condition on ordering frames:

(Universality) the field S_i of $\angle_i$ is always the set of all worlds.

I know of no very strong reasons for or against imposing the requirement of Universality, and accordingly I shall treat it as an optional extra. Suppose we are given an ordering frame that does not satisfy Universality, and suppose we would prefer one that does. The natural adjustment is as follows. Where the original frame assigns to i the ordering $\angle_i$ of S_i, let the new frame assign the ordering $\angle_i^+$ of all worlds, where $j \angle_i^+ k$ iff either $j \angle_i k$ or j does and k does not belong to S_i. Call the new frame the *universalisation* of the original frame. The difference between the two frames only matters when the antecedent of a counterfactual is true at some worlds, but not at any of the worlds in S_i. Then the original frame treats the counterfactual as vacuous, making it true at i regardless of its consequent; whereas the universalisation makes it true at i iff the consequent is true at every world where the antecedent is, so that the antecedent stricly implies the consequent.

Some further notation will prove useful. Let us write $\langle \angle_i \rangle$ for the ordering frame that assigns to any world i the ordering $\angle_i$ (S_i can be left out, since it is the field of $\angle_i$). Let us also write $j \trianglelefteq_i k$ to mean that either $j \angle_i k$ or j is identical to k and belongs to S_i. The relation $\trianglelefteq_i$ is then a nonstrict partial ordering of S_i. And let us write $\sim_i$ to mean that neither $j \angle_i k$ nor $k \angle_i j$ although both j and k belong to S_i. If so, j and k are either identical, or tied as differing equally from i, or incomparable with respect to their difference from i. The relation $\sim_i$ is reflexive and symmetric, but not in general transitive.

3. PREMISE SEMANTICS IN THE FINITE CASE

A simpler way to think of factual background is as a set of facts – a set of true propositions about the world, which together serve to distinguish it from all other worlds. These facts serve as auxiliary premises which may join with the antecedent of a counterfactual to imply the consequent, thereby making the counterfactual true against that factual background. The obvious problem is that some of the facts will contradict the antecedent (unless it is true), so the entire set of them will join with the antecedent to imply anything whatever. We must therefore use subsets of the factual premises, cut down just enough to be consistent with the antecedent. But how shall we cut the premise set down – what goes, what stays? We might invoke some system of weights or priorities, telling us that some of the factual premises are to be given up more readily than others. That would lead directly back to ordering semantics. Alternatively, we might treat the premises equally. We might require that the cut-down premise set must join with the antecedent to imply the consequent no matter how the cutting-down is done; all ways must work. That is the approach to counterfactuals taken by Kratzer in [4]; it is a special case of her treatment of conditional modality in [3], which in turn is based on her treatment of modality in [2].

We must be selective in the choice of premises. If we take all facts as premises, then (as is shown in [4]) we get no basis for discrimination among the worlds where an antecedent holds and the resulting truth condition for counterfactuals is plainly wrong. By judicious selection, we can accomplish the same sort of discrimination as would result from unequal treatment of premises. As Kratzer explains in [4], the outcome depends on the way we lump items of information together in single premises or divide them between several premises. Lumped items stand or fall together, divided items can be given up one at a time. Hence if an item is lumped into several premises, that makes it comparatively hard to give up; whereas it it is confined to a premise of its own, it can be given up without effect on anything else. This lumping and dividing turns out to be surprisingly powerful as a method for discriminating among worlds – so much so that, as will be shown, premise semantics can do anything that ordering semantics can. Formally, there is nothing to choose. Intuitively, the question is whether the same premises that it would seem natural to select are the ones that lump and divide properly; on that question I shall venture no opinion.

Let us identify a proposition with the set of worlds where it holds. Logical terminology applied to propositions has its natural set-theoretic meaning: conjunctions and disjunctions are intersections and unions, the necessary and impossible propositions are the set of all worlds and the empty set, consistency among propositions is nonempty intersection, implication is inclusion of the intersection of the premises in the conclusion, and so on. Facts about a world i are sets of worlds that contain i, and a set of facts rich enough to distinguish i from all other worlds is one whose intersection has i as its sole member.

Again we distinguish the contribution of the world from the contribution of a 'frame' determined somehow by linguistic practice and context. The world provides the facts, the frame selects some of those facts as premises. A counterfactual is evaluated at a world, and according to a frame.

We define a *premise frame* $\langle H_i \rangle$ as a function that assigns to any world i a set H_i of propositions – *premises* for i – satisfying the condition:

(Centering) i does, and all other worlds do not, belong to every proposition in H_i.

An *A-consistent premise set* for i is a subset of H_i that is consistent with the proposition A; and it is a *maximal A-consistent premise set* for i iff, in addition, it is not properly included in any larger A-consistent premise set for i. We can lay down the following truth condition for a counterfactual from A to C: it is true at world i, according to a precise frame, iff

(PF) whenever J is a nonempty maximal A-consistent premise set for i, J and A jointly imply C.

This is almost, but not quite, Kratzer's truth condition for the finite case; hers is obtained by deleting the word 'nonempty'. The reason for the change is explained below.

Worlds where none of the premises in H_i hold are ignored in evaluating counterfactuals at i, just as worlds outside S_i are in ordering semantics. If the antecedent of a counterfactual holds only at such ignored worlds, it is just as if the antecedent holds at no worlds at all: there are no nonempty maximal A-consistent premise sets, so the counterfactual is true regardless of its consequent. If we find it objectionable that some worlds are thus ignored, we could do as Kratzer does and delete the word 'nonempty', so that the counterfactual will be true at i iff A (jointly with the empty set) implies C. Alternatively, we could impose an extra condition on premise frames to stop the problem from arising:

(Universality) every world belongs to some proposition in H_i.

I shall treat this requirement of Universality as an optional extra, like the corresponding requirement of Universality in ordering semantics.

Suppose we have a premise frame $\langle H_i \rangle$ that does not satisfy Universality, and we want one that does; then we can simply take a new frame $\langle H_i^+ \rangle$ that assigns to any world i the set consisting of all the propositions in H_i and the neccessary proposition as well. Call $\langle H_i^+ \rangle$ the *universalisation* of $\langle H_i \rangle$. Except in the case considered, in which the antecedent holds only at some worlds outside all the premises for i, a premise frame and its universalisation evaluate all counterfactuals alike. In effect, Kratzer opts for Universality; but she builds it into her truth condition instead of imposing it as an extra requirement on premise frames. We are free to start with a frame that does not satisfy Universality, but we universalise it whenever we use it: a counterfactual is true at a world according to $\langle H_i \rangle$ under Kratzer's truth condition iff it is true at that world according to $\langle H_i^+ \rangle$ under the neutral truth condition PF. I think it better to use PF, both for the separation of distinct questions and for easy comparison with extant versions of ordering semantics. If we want to consider exactly Kratzer's version of premise semantics, we need only impose Universality as a requirement on frames.

4. EQUIVALENCE OF FRAMES

Given a premise frame $\langle H_i \rangle$, there is a natural way to derive from it an ordering frame $\langle \angle_i \rangle$: let S_i be the union of the propositions in H_i; and for any j and k in S_i, let $j \angle_i k$ iff all propositions in H_i that hold at k hold also at j, but some hold at j that do not hold also at k. The worlds that can be ordered are those where at least some of the premises hold; a closer world conforms to all the premises that a less close world conforms to and more besides. If each $\angle_i$ is derived in this way from the corresponding H_i, it is easily seen that $\langle \angle_i \rangle$ must be an ordering frame. Let us call the frames $\langle H_i \rangle$ and $\langle \angle_i \rangle$ *equivalent*.

Equivalent frames evaluate counterfactuals alike, at least in the finite case. Let $\langle H_i \rangle$ and $\langle \angle_i \rangle$ be equivalent frames. Then for any propositions A and C and any world i, PF holds iff OF holds.

Proof. Let j be any A-world and let J be the set of all propositions in H_i that hold at j. It is enough to show that (i) J is a nonempty maximal A-consistent premise set for i iff (ii) j is a closest A-world to i. We may assume that J is nonempty, else (i) and (ii) are both false. ($\rightarrow$) If not (ii), we have $k \in A \cap S_i$ such that $k \angle_i j$. Let K be the set of propositions in H_i that hold at k. K is an A-consistent premise set for i and it properly includes J, so not (i). ($\leftarrow$) If not (i), J must be properly included in some larger A-consistent premise set for i, call it K. Take any k in $A \cap \bigcap K$. Then $k \angle_i j$, so not (ii). Q.E.D.

By definition, every premise frame is equivalent to some unique ordering frame. However, two premise frames may be equivalent to the same ordering frame. Suppose two premise frames are alike except that one assigns to i the premises $\{i\}$ and $\{i, j, k\}$ while the second assigns to i the premises $\{i, j\}$ and $\{i, k\}$. Either way, the derived ordering $\angle_i$ is the same: S_i is $\{i, j, k\}$, $i \angle_i j$, $i \angle_i k$, and $j \sim_i k$. This means that premise frames contain surplus information – information that makes no difference to the way the premise frames do their job of evaluating counterfactuals. Intuitively, this surplus information concerns the difference between ties and incomparabilities. The first of our frames represents j and k as alike in the way they differ from i, whereas the second represents them as departing from i in different directions. Ordering frames – if they use strict orderings, as in my present formulation – omit this surplus information. Any two of them disagree in their evaluation of some counterfactual at some world, and from the fact that $j \sim_i k$ we cannot tell whether to regard j and k as tied or as incomparable.

Every ordering frame can be derived from an equivalent premise frame. Thus the equivalence of premise and ordering frames is a many-one correspondence, exhausting both classes.

Proof. Suppose given an ordering frame $\langle \angle_i \rangle$. For each world i, let H_i be $\{\{j: j \trianglelefteq_i k\}: k \in S_i\}$. Centering for $\langle H_i \rangle$ follows from Centering for $\langle \angle_i \rangle$: any i belongs to each member of H_i, and $\{i\}$ belongs to H_i since it is $\{j: j \trianglelefteq_i i\}$, so $\cap H_i$ is $\{i\}$. Hence $\langle H_i \rangle$ is a premise frame. Further, $\langle H_i \rangle$ and $\langle \angle_i \rangle$ are equivalent. Each S_i is $\cup H_i$: for any $k \in S_i$ we have $k \in \{j: j \trianglelefteq_i k\} \in H_i$, and for any $j \in \cup H_i$ we have $j \in S_i$ since $j \trianglelefteq_i k \in S_i$. Also, for any g and h in S_i, $g \angle_i h$ iff g belongs to all the members of H_i that h belongs to and more besides. ($\rightarrow$) If h belongs to $\{j: j \trianglelefteq_i k\}$, then so does g since $g \angle_i h \trianglelefteq_i k$. But g belongs to $\{j: j \trianglelefteq_i g\}$ and h does not, and $g \in S_i$. ($\leftarrow$) Since $h \in S_i$ and h belongs to $\{j: j \trianglelefteq_i h\}$, so does g. But $g \neq h$ since they do not belong to exactly the same sets, so $g \angle_i h$. Q.E.D.

Finally, it is immediate from the definition of equivalence that if $\langle H_i \rangle$ and $\langle \angle_i \rangle$ are equivalent frames, then the former satisfies the optional Universality requirement of premise semantics iff the latter satisfies the optional Universality requirement of ordering semantics. Further, any two equivalent frames have equivalent universalisations.

5. PARTIAL VERSUS MULTIPLE ORDERINGS

The results of Section 4 show that premise semantics is equivalent to a version of ordering semantics – Pollock's version, if we ignore questions that arise in the infinite case – in which the orderings may be merely partial. Kratzer joins Pollock, and seemingly disagrees with Stalnaker and Lewis, in permitting worlds to differ from a given world in incomparable ways. However, I shall argue that there is less to this disagreement than meets the eye.

Although an ordering frame cannot, and need not, distinguish incomparabilities from ties, it can nevertheless reveal that incomparabilities are present. Recall that the relation $\sim_i$ is reflexive and symmetric. Unless it is transitive as well, some cases in which $j \sim_i k$ must be incomparabilities, not ties or identities; for the relation of being tied or identical must surely be an equivalence relation. If $\sim_i$ is transitive, on the other hand, there is no bar to regarding it as the relation of being tied or identical. Let us do so. To prohibit incomparabilities, as Lewis and Stalnaker do, is to impose the following as an extra requirement on ordering frames:

(Comparability) $\sim_i$ is transitive.

That means that each $\angle_i$ must be at least a strict weak ordering. (Lewis's formulation, in [5]: 48–50, actually uses nonstrict weak orderings, but it is equivalent to the formulation considered here.) To prohibit ties as well, as Stalnaker does, is to impose the stronger requirement that $\sim_i$ is just the relation of identity among worlds in S_i, or equivalently:

(Trichotomy) for any j and k in S_i, $j \angle_i k$ or $j = k$ or $k \angle_i j$.

That means that each $\angle_i$ is a strict simple ordering.

Pollock [7] argues that no ordering frame without incomparabilities could give the intuitively correct evaluations of all the counterfactuals in a certain small set. But Pollock's argument is suspect, as noted by Loewer [6]: 111. Pollock's example involves English counterfactual sentences with seemingly disjunctive antecedents; such sentences behave oddly in a way that would account for Pollock's evidence with no need for any incomparabilities.

A better reason to question Comparability, however, is close at hand. Ordinary counterfactuals usually require only the comparison of worlds with a great deal in common, from the standpoint of worlds of the sort we think we might inhabit. An ordering frame that satisfies Comparability would be cluttered up with comparisons that matter to the evaluation of counterfactuals only in peculiar cases that will never arise. Whatever system of general principles we use to make the wanted comparisons will doubtless go on willy nilly to make some of the unwanted comparisons as well, but it seems not likely that it will settle them all (not given that it makes the wanted comparisons in a way that fits our counterfactual judgements). An ordering frame that satisfies Comparability would be a cumbersome thing to keep in mind, or to establish by our linguistic practice. Why should we have one? How could we? Most likely we don't.

This argument is persuasive, and I know of no one who would dispute it. However, it is not exactly an argument against Comparability. Rather, it is an argument against the combination of Comparability with full determinacy of the ordering frame. We need no partial orderings if we are parepared to admit that we have not bothered to decide quite which total orderings (weak or simple, as the case may be) are the right ones. The advocates of Comparability certainly are prepared to admit that the ordering frame is left underdetermined by linguistic practice and context. (Stalnaker [9]; Lewis [5]: 91–95.) So where Pollock sees a determinate partial ordering that leaves two worlds incomparable, Stalnaker and Lewis see a multiplicity of total orderings that disagree in their comparisons of the two worlds, with nothing in practice and context to select one of these orderings over the rest. Practice and context determine a class of frames each satisfying Comparability, not a single frame that fails to satisfy Comparability.

Formally, there is nothing to choose, so long as we are concerned only with conditions of determinate truth for counterfactuals, and so long as we stick to the finite case. Ordering semantics with and without Comparability, or even Trichotomy, are in a sense equivalent. Premise semantics is in the same sense equivalent not only to Pollock's version of ordering semantics but also to Lewis's, and even to Stalnaker's.

Compare the difference between Lewis and Stalnaker: where Lewis sees a tie, Stalnaker sees indeterminacy between simple orderings that break that tie in opposite ways. A counterfactual is true on Lewis's semantics iff it is true on Stalnaker's semantics no matter how the ties are broken. (See Lewis [5]: 81–83; and for a fuller discussion, van Fraassen [10].) The same method of reconciliation applies also to the seeming disagreement about Comparability. (We shall consider ordering semantics only, but of course what follows carries over to premise semantics by equivalence of frames.) Where Pollock or Kratzer sees an incomparability, Lewis or Stalnaker sees indeterminacy between total (weak or simple) orderings that make the missing comparison in all possible ways. A counterfactual is true on Pollock's or Kratzer's semantics iff it is true on Lewis's or Stalnaker's semantics no matter how the missing comparisons are made.

Let us call frame $\langle \angle_i^* \rangle$ a *refinement* (alternatively, a *Stalnaker refinement*) of frame $\langle \angle_i \rangle$ iff (i) for any i, S_i^* is the same as S_i, (ii) whenever $j \angle_i k$, then

$j \angle_i^* k$, and (iii) $\langle \angle_i^* \rangle$ satisfies Comparability (alternatively, Trichotomy). We wish to show that a counterfactual is true at a world according to $\langle \angle_i \rangle$ iff it is true at that world according to every refinement (alternatively, every Stalnaker refinement) of $\langle \angle_i \rangle$, on truth condition OF. (Here I assume that the antecedent and consequent will express the same proposition according to the original frame and the refinements; that may not be so if they are themselves counterfactuals, or compounded in part from counterfactuals.)

Proof. It suffices to show that (i) j is a closest A-world to h according to some refinement (alternatively, some Stalnaker refinement) $\langle \angle_i^* \rangle$ of $\langle \angle_i \rangle$ iff (ii) j is a closest A-world to h according to $\langle \angle_i \rangle$. Assume $j \in A \cap S_h$, else (i) and (ii) are both false. ($\rightarrow$) If not (ii), we have $k \in A$ such that $k \angle_h j$. Then also $k \angle_h^* j$ for any refinement (alternatively, any Stalnaker refinement) $\langle \angle_i^* \rangle$ of $\langle \angle_i \rangle$, so not (i). ($\leftarrow$) We will construct a Stalnaker refinement $\langle \angle_i^* \rangle$ of $\langle \angle_i \rangle$ such that whenever $j \sim_h k$ and $j \neq k$, then $j \angle_h^* k$. Suppose for *reductio* that j is not a closest A-world to h according to $\langle \angle_i^* \rangle$. Then we have $k \in A$ such that $k \angle_h^* j$. Not $k \angle_h j$ by (ii). Not $j \angle_h k$ since then $j \angle_h^* k \angle_h^* j$. Not $j = k$ since then $k \angle_h^* k$. Not both $j \sim_h k$ and $j \neq k$ since then again $j \angle_h^* k \angle_h^* j$. But there is no other alternative, so the supposition is refuted. It remains to construct the refinement $\langle \angle_i^* \rangle$. If $i \neq h$, let $\angle_i^0 = \angle_i$. Let $f \angle_h^0 g$ iff either $f \angle_h g$ or there is some k such that $j \sim_h k, j \neq k, f \stackrel{\angle}{=}_h j$, and $k \stackrel{\angle}{=}_h g$. Now for each i, take an arbitrary sequence $\langle f_0, g_0 \rangle$, $\langle f_1, g_1 \rangle, \ldots$ of all pairs of worlds in S_i, and form a parallel sequence $\angle_i^0$, $\angle_i^1, \ldots$ as follows. We have $\angle_i^0$. If $f_n \angle_i^n g_n$ or if $f_n = g_n$ or if $g_n \angle_i^n f_n$, let $\angle_i^{n+1} = \angle_i^n$. Otherwise, let $f \angle_i^{n+1} g$ iff either $f \angle_i^n g$ or both $f \stackrel{\angle}{=}_i^n f_n$ and $g_n \stackrel{\angle}{=}_i^n g$. Let $\angle_i^*$ be the last term of this sequence. (In the infinite case, invoke the axiom of choice to take the pairs in an arbitrary transfinite sequence; and in forming the parallel sequence, take unions at limit ordinals and at the end.) Transitivity and asymmetry are preserved at each step, and in the preliminary step from $\angle_h$ to $\angle_h^0$ (and in taking unions, in the inifinite case), so each $\angle_i^*$ is (at least) a strict partial ordering. At each step the field of the ordering remains S_i. Also $\angle_i$ is included in $\angle_i^*$, since we only add pairs and never remove any. Hence $\langle \angle_i^* \rangle$ satisfies Centering, and so is a frame. Also any pair $\langle f, g \rangle$ will be added at some step unless $\langle g, f \rangle$ is present first or $f = g$, so $\langle \angle_i^* \rangle$ satisfies Trichotomy and is a Stalnaker refinement of $\langle \angle_i \rangle$. Finally, whenever $j \sim_h k$ and $j \neq k$, $j \angle_h^0 k$ and hence $j \angle_h^* k$. Q.E.D.

Although there is no remaining issue about conditions of determinate truth for counterfactuals in the finite case – determinate truth being truth according to all frames within the range of indeterminacy – some lesser questions remain in dispute. What are we to say of a counterfactual that is false according to the original frame, true according to some of its refinements, and false according to other refinements? Is it false or is it indeterminate in truth value? What are we to say of its negation? What of a disjunction of counterfactuals such that the original frame makes neither disjunct true, but every refinement makes one or the other disjunct true? I think these questions are best asnswered by a version of ordering semantics that prohibits all incomparabilities but permits at least some ties, however I shall not argue that case here.

6. THE INFINITE CASE

Our definitions and results all carry over to the infinite case. However, the adequacy of the truth conditions OF and PF becomes doubtful. For all we have said so far, there might be infinite sequences without limit: in ordering semantics, ever-closer A-worlds to i but no closest one, or in premise semantics, ever-larger A-consistent premise sets for i but no maximal one. Then OF and PF give the absurd result that any counterfactual whose antecedent expresses A is true at i regardless of its consequent.

There is a choice between two remedies, and on this question we change partners. Lewis and Kratzer prefer to modify the truth condition, so that we get reasonable evaluations even in these troublesome cases. We shall consider shortly how this may be done. Stalnaker and Pollock prefer not to modify the truth condition, but rather to make sure that the troublesome cases never arise. They impose an extra requirement on ordering frames:

(Limit Assumption) unless no A-world belongs to S_i, there is some closest A-world to i.

We could likewise impose an extra requirement on premise frames:

(Limit Assumption) unless A is consistent with no proposition in H_i, there is some nonempty maximal A-consistent premise set for i.

It follows immediately from our first result in Section 4 that these two formulations agree: A at i violates the Limit Assumption for a premise frame iff A at i violates the Limit Assumption for the equivalent ordering frame. Hence one of two equivalent frames satisfies the Limit Assumption iff the other does.

The Limit Assumption makes infinite frames no more troublesome than finite ones. It makes it safe to continue to use our original truth conditions OF and PF, which certainly are simpler and more intuitive than the modified versions of Lewis and Kratzer. It has the further advantage, regarded as decisive by Pollock [7] and Hertzberger [1], of validating certain plausible principles of the infinitary logic of counterfactuals:

(Consistency Principle) if $C_1, C_2, \ldots$ are inconsistent, then so are the counterfactuals from A to $C_1, C_2, \ldots$ (except that they may be vacuously true together, in which case all counterfactuals from A are true);

(Consequence Principle) if $C_1, C_2, \ldots$ jointly imply B, then the counterfactuals from A to $C_1, C_2, \ldots$ jointly imply the counterfactual from A to B.

These principles can fail under the modified truth conditions of Lewis and Kratzer when the set of C_n's is infinite and there is a violation of the Limit Assumption.

Despite these formal advantages, I still think it best not to impose the Limit Assumption. The trouble with it is that it is apt to conflict with any account we might wish to give of how the orderings or the premise sets that comprise a frame are determined. Conflicts can arise in a variety of ways. The example of the line, given in [5]: 20–21, shows how the Limit Assumption may fail if we base our orderings on a weighted sum of degrees of similarity or dissimilarity in various respects. Pollock [7] draws the conclusion that we should not construct the orderings that way – he puts his point by saying that they are not similarity orderings – but I think he underestimates the problem. The Limit Assumption can fail in quite different ways also. It can fail even if we stick to atomistic, all-or-nothing respect of similarity and difference, and even if we give up Comparability rather than balancing these off against one another.

Example. Consider a premise frame that assigns to world i an infinite set H_i of independent propositions, so that any conjunction of some of these propositions with the negations of all the rest is consistent. Consider also the equivalent ordering frame, according to which $j \angle_i k$ iff j conforms to all the premises in H_i that k conforms to and more besides. Thus $\angle_i$ is rife with incomparabilities. These frames fail to satisfy the Limit Assumption. Let A hold at exactly those worlds where infinitely many propositions in H_i fail to hold. A premise set for i is A-consistent iff it leaves out infinitely many members of H_i, and there is no maximal such set. Likewise there is no closest A-world to i.

If we want the Limit Assumption, I take it that what we need is some sort of coarse-graining. We must imitate the finite case by ignoring most of the countless respects of difference that make the possible worlds infinite in number. Coarse-graining is certainly a formal option; whether it can be built into an intuitively adequate analysis of counterfactuals seems to me to be an open question. Therefore I think it best to remain neutral on the Limit Assumption, and to replace truth conditions OF and PF by modified versions that do not need the Limit Assumption to work properly.

Lewis's version of ordering semantics uses a truth condition, given in [5]: 49, which does not need the Limit Assumption: a counterfactual from A to C is true at world i, according to an ordering frame, iff

(OC) unless no A-world belongs to S_i, there is some A-world j in S_i such that for any A-world k in S_i, either C holds at k or $j \angle_i k$.

But OC is not satisfactory for our present purposes. Although it does not need the Limit Assumption, it does need Comparability. It might mis-evaluate a counterfactual as false – say, one that is true under OF in the finite case – because the worlds where the antecedent holds divide into incomparable classes.

We want a fully neutral truth condition: one that needs neither the Limit Assumption nor Comparability. I propose the following: a counterfactual from A to C is true at world i, according to an ordering frame, iff

(O) for any A-world h in S_i, there is some A-world j such that (i) $j \trianglelefteq_i h$, and (ii) C holds at any A-world k such that $k \trianglelefteq_i j$.

This condition is fully neutral. However, it is a generalization both of OF and of OC, reducing to the former given the Limit Assumption and to the latter given Comparability.

Proof. Assume that $A \cap S_i$ is nonempty, else O, OF, and OC all are true. (1) O implies OF. Let h be any A-world closest to i. By O we have $j \in A$ such that (i) $j \leqq_i h$, and hence $j = h$, and (ii) C holds at any A-world k such that $k \leqq_i j$, and hence at h itself. (2) OF and the Limit Assumption imply O. Given $h \in A \cap S_i$, let B be the set of all worlds $g \in A$ such that $g \leqq_i h$. By the Limit Assumption, since $h \in B \cap S_i$, we have j which is a closest B-world to i. Since $j \in B, j \leqq_i h$. Also j is a closest A-world to i, for if $f \in A$ and $f \angle_i j$, then $f \in B$ and so j is not a closest B-world. So $j \in C$ by OF. Whenever $k \in A$ and $k \leqq_i j$, it must be that $k = j$ and hence $k \in C$. (3) O and Comparability imply OC. By O, since $A \cap S_i$ is nonempty, we have some $j \in A \cap S_i$ such that whenever $k \in A$ and $k \leqq_i j$ then $k \in C$. If for any $k \in A \cap S_i$ either $k \in C$ or $j \angle_i k$, we are done. If not, we have $g \in A \cap S_i$ such that neither $g \in C$ nor $j \angle_i g$. Not $g \leqq_i j$, else $g \in C$, so $g \sim_i j$. By O again, we have $j' \in A$ such that (i) $j' \leqq_i g$, and (ii) whenever $k \in A$ and $k \leqq_i j'$ then $k \in C$, so in particular $j' \in C$. Then $j' \neq g$ so $j' \angle_i g$. By Comparability, $j' \angle_i j$. For any $k \in A \cap S_i$, either $j' \angle_i k$, or $k \sim_i j'$ in which case by Comparability $k \angle_i j$ and so $k \in C$, or $k \angle_i j'$ in which case again $k \angle_i j$ and so $k \in C$. (4) OC implies O. By OC we have $g \in A \cap S_i$ such that for any $k \in A \cap S_i$ either $k \in C$ or $g \angle_i k$. For any $h \in A \cap S_i$, let $j = g$ if $g \leqq_i h, j = h$ otherwise. Either way, (i) $j \leqq_i h$, and (ii) whenever $k \in A$ and $k \leqq_i j$, not $g \angle_i k$ so $k \in C$. Q.E.D.

As a neutral truth condition for premise semantics, we can take the following: a counterfactual from A to C is true at i, according to a premise frame, iff

(P) For any nonempty A-consistent premise set H for i, there is some A-consistent premise set J for i such that (i) H is included in J, and (ii) J and A jointly imply C.

This is almost, but not quite, Kratzer's truth condition in [4] for the infinite case. As in the finite case, hers is obtained by deleting the word 'nonempty'; it has Universality built in, so that a counterfactual is true at a world according to a frame $\langle H_i \rangle$ under Kratzer's truth condition iff it is true at that world according to the universalization $\langle H_i^+ \rangle$ under truth condition P.

When we switch to the neutral truth conditions O and P, it remains true that equivalent frames evaluate counterfactuals alike. (From that and previous results, it follows that P reduces to PF given the Limit Assumption, as we would wish.)

Let $\langle H_i \rangle$ and $\langle \angle_i \rangle$ be equivalent frames. Then for any propositions A and C and any world i, P holds iff O holds.

Proof. Assume that there is some nonempty A-consistent premise set for i, and hence some world in $A \cap S_i$; else P and O both are false. ($\rightarrow$) Let h be any world in $A \cap S_i$, and let H be the set of propositions in H_i that hold at h. H is a nonempty A-consistent premise set for i. By P we have an A-consistent premise set J for i such that (i) $H \subseteq J$, and (ii) J and A jointly imply C. If $J = H$ let $j = h$, otherwise let j be any world in $A \cap \bigcap J$. Either way, $j \in A$ and $j \leqq_i h$. Whenever $k \in A$ and $k \leqq_i j$, $k \in \bigcap J$ so $k \in C$. ($\leftarrow$) Let H be any nonempty A-consistent premise set for i, and let h be any world in $A \cap \bigcap H$. Then $h \in A \cap S_i$, so by O we have $j \in A$ such that (i) $j \leqq_i h$, and (ii) whenever $k \in A$ and $k \leqq_i j$ then $k \in C$. Let J be the set of propositions in H_i that hold at j. Whether $j = h$ or $j \angle_i h$, $H \subseteq J$. J is an A-consistent premise set for i. So if J and A jointly imply C, we are done. If not, there is some world $h' \in A \cap \bigcap J$ such that $h' \notin C$. Since $h' \in A \cap S_i$, by O we have $j' \in A$ such that (i) $j' \leqq_i h'$, and (ii) whenever $k \in A$ and $k \leqq_i j'$ then $k \in C$, so in particular $j' \in C$ and hence $j' \neq h'$. Then $j' \angle_i h'$. Let J' be the set of propositions in H_i that hold at j'; $H \subseteq J \subseteq J'$ and $J \neq J'$. J' is an A-consistent premise set for i. Consider any $k \in A \cap \bigcap J'$; $k \angle_i j$, so $k \in C$. Hence J' and A jointly imply C. Q.E.D.

Unfortunately, on thing we lose in switching to our modified truth conditions is the reconciliation about Comparability that we considered in Section 5. Under O, we can have a counterfactual true at a world according to a frame, but not according to all refinements of that frame.

Example. Let the A-worlds in S_i be indexed by the integers: $\ldots, j_{-2}, j_{-1}, j_0, j_1, j_2, \ldots$. Let $j_m \angle_i j_n$ iff m is even and $n = m + 1$; let $j_m \angle_i^* j_n$ iff $m \angle n$ in the usual ordering of the integers; and let C hold at j_m iff m is even. Then under O the counterfactual from A to C is true at i according to a frame that assigns $\angle_i$ to i, but not according to a refinement that assigns $\angle_i^*$.

Perhaps some more complicated version of the reconciliation would succeed, but I fear the complication would make it at least somewhat *ad hoc*.

7. FRAMES WITHOUT CENTERING

We could remove the Centering requirements from our definitions of ordering and premise frames, thereby redefining them in a more general sense. Generalized ordering frames are discussed in Lewis [5]: 97–121, and generalized premise frames in Kratzer [2] and [3].

By dropping Centering, we can extend ordering or premise semantics to certain relatives of counterfactuals, notably deontic conditionals. These too are evaluated against a background that may be expressed by orderings or premise sets. However, the background is deontic rather than factual: we may say that (from the standpoint of a certain world, and according to a frame somehow determined by our linguistic practice and by context) one world is better than another, or a certain premise is something that ought to hold. Lewis discusses ordering semantics for deontic conditionals in [5]: 96–104, and Kratzer's premise semantics for conditional modality in [3] applies to deontic conditionals as well as to counterfactuals. Formally, except for Centering, the ordering or premise frames and truth conditions given for deontic conditionals are like those for counterfactuals. Similar questions can arise about Universality, Comparability and Trichotomy, and the Limit Assumption versus modified truth conditions for the infinite case. Though we certainly do not want Centering, we might – but we might not – want some of its consequences, such as a requirement that all premise sets are consistent.

Our equivalence results did not depend on Centering, so they carry over to generalized ordering and premise frames. Thus premise semantics for deontic conditionals is again equivalent to a version of ordering semantics that permits merely partial orderings, and again partial orderings can in the finite case be replaced by classes of weak or simple orderings.

In fact, we made almost no use of Centering in our proofs. None, except in checking that certain structures constructed from frames that satisfied Centering were themselves frames satisfying Centering. We can now restate those results: (i) if a generalized premise frame and a generalized ordering frame are equivalent, both or neither satisfies Centering; (ii) if a generalized ordering frame is a refinement of a frame that satisfies Centering, then it too satisfies Centering.

Even in the case of ordering and premise semantics for counterfactuals, Centering might be questioned; see Lewis [5]: 28. It is clear that we must not have worlds closer to i than i itself, or premises for i that do not hold at i. It is less clear than we must not have other worlds that differ not at all from i, or where all the premises for i hold. Though I am on the whole persuaded to require Centering – in company with Stalnaker, Pollock, and Kratzer – it is worth noting the possibility of versions of ordering and premise semantics for counterfactuals in which Centering is weakened.

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